## Name:

## Section $4.1 \& 4.2$ - In class examples

Math 151 - Spring 2018

## Section 4.1

1. Find and classify the critical points of $x^{3}-9 x^{2}-48 x+52$ using the second derivative test. $f^{\prime}(x)=3 x^{2}-18 x-48=3(x-8)(x+2)$ setting $f^{\prime}(x)=0$ and solving yeilds $x=8,-2$. We can test the nature of the critical points using the second derivative. Indeed, $f^{\prime \prime}(x)=6 x-18$ so then $f^{\prime \prime}(8)=30$ and $f^{\prime \prime}(-2)=-30$. We conclude that $x=8$ is a local minimum and $x=-2$ is a local maximum.
2. The value of an investment at time $t$ is given by $S(t)$. The rate of change, $S^{\prime}(t)$, of the value of the investment is shown in the figure below

(a) What are the critical points of the function $S(t)$. The critical points of $S$ occur at times t when $S^{\prime}(t)=0$. We see that $S^{\prime}(t)=0$ at $t=1,4$, and 6 , so the critical points occur at $t=1,4$, and 6.
(b) Identify each critical point as a local maximum, a local minimum, or neither.

We see that $S^{\prime}(t)$ is positive to the left of 1 and between 1 and 4 , that $S^{\prime}(t)$ is negative between 4 and 6 , and that $S^{\prime}(t)$ is positive to the right of 6 . Therefore $(t)$ is increasing to the left of 1 and between 1 and 4 (with a slope of zero at 1), decreasing between 4 and 6 , and increasing again to the right of 6 . We see that $S$ has neither a local maximum nor a local minimum at the critical point $t=1$, but that it has a local maximum at $t=4$ and a local minimum at $t=6$.
(c) Explain the financial significance of each of the critical points.

At time $t=1$ the investment momentarily stopped increasing in value, though it started increasing again immediately afterward. At $t=4$, the value peaked and began to decline. At $t=6$, it started increasing again.

