## Name:

## Homework 4 solutions

Math 151, Applied Calculus, Spring 2018
Section 2.1 - 1,4,5,6,9,11,12,18

1. (a) $f^{\prime}(1950)$ is negative. That means that the number of farms in the US was decreasing in 1950.
(b) The function $N=f(t)$ is decreasing in 1960 as well as in 1980 but it is decreasing faster in 1960 than in 1980. Therefore, $f^{\prime}(1960)$ is more negative than $f^{\prime}(1980)$.
2. (a) (i) $8.4 \mathrm{~m} / \mathrm{sec}$, (ii) $8.04 \mathrm{~m} / \mathrm{sec}$, (iii) $8.004 \mathrm{~m} / \mathrm{sec}$.
(b) We see in part (a) that as we choose a smaller and smaller interval around $t=1$ the average velocity appears to be getting closer and closer to 8 , so we estimate the instantaneous velocity at $t=1$ to be $8 \mathrm{~m} / \mathrm{sec}$.
3. See in-class example.
4. (a) $7 \mathrm{ft} / \mathrm{sec}$.
(b) $4 \mathrm{ft} / \mathrm{sec}$.
5. 5.549 using an interval $x=1$ to $x=1.001$.
6. The slope is positive at A and D ; negative at C and F . The slope is most positive at A ; most negative at F.

| Slope | -3 | -1 | 0 | $1 / 2$ | 1 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Point | F | C | E | A | B | D |

12. 
13. (a) $f(7)=3$.
(b) $f^{\prime}(7)=4$

## Section 2.2 - 9,10,12,18-21

9. (a) $x_{3}$, (b) $x_{4}$, (c) $x_{5}$ (d) $x_{3}$

| $x$ | 0 | 5 | 10 | 15 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f^{\prime}(x)$ | -6 | -3 | -1.8 | -1.2 | -1.2 |

10. 
11. $f^{\prime}(x)$ should be a constant function.
12. VIII
13. IV
14. II
15. VI

## Section $2.3-7,10,14,15,16,17,22,28,30,31,42,43$

7. (a) The 12 represents the weight of the chemical; therefore, its units are pounds. The 5 represents the cost of the chemical;therefore, its units are dollars. The statement $f(12)=5$ means that when the weight of the chemical is 12 pounds, the cost is 5 dollars.
(b) We expect the derivative to be positive since we expect the cost of the chemical to increase when the weight bought increases.
(c) Again, 12 is the weight of the chemical in pounds. The units of the 0.4 are dollars/pound since it is the rate of change of the cost as a function of the weight of the chemical bought. The statement $f^{\prime}(12)=0.4$ means that the cost is increasing at a rate of 0.4 dollars per pound when the weight is 12 pounds, or that an additional pound will cost about an extra 40 cents.
8. Rate of change of revenue $\approx \frac{\$ 5.5 \text { million }}{0.1 \text { point }}=\$ 55$ milliondollars $/$ point so $f^{\prime}(4.3) \approx 55$.
9. (a). This means that investing the $\$ 1000$ at $5 \%$ would yield $\$ 1649$ after 10 years.
(b). $g^{\prime}(5)=165$ means that at $5 \%$ interest, would increase by about $\$ 165$ if the interest rate were increased by $1 \%$
10. (a). The derivative $f^{\prime}(t)$ appears to be negative for most of the period 2005-2009, because according to the table, gold production is decreasing. There appears to have been a recovery during 2008 - 2009 because production has increased so $f^{\prime}(t)$ appears to be positive during this period.
(b). The derivative (or rate of change) appears to be greatest between 2008 and 2009
(c). $f^{\prime}(2009) \approx 160$ metric tons/year.
(d). 2610, 3410 .
11. (a). $f(1800)=155$ means that consuming 1800 Calories per day results in a weight of 155 pounds.
(b). $f^{\prime}(2000)=0$ means that consuming 2000 Calories per day causes neither weight gain nor loss.
(c). pounds/(Calories/day).
12. (a). The statement $f(200)=1300$ means that it costs $\$ 1300$ to produce 200 gallons of the chemical.
(b). The statement $f^{\prime}(200)=6$ means that when the number of gallons produced is 200 , costs are increasing at a rate of $\$ 6$ per gallon. In other words, it costs about $\$ 6$ to produce the next (the 201 st) gallon of the chemical.
13. $3.4,2.6$
14. (a). The statement $f(8)=5.1$ means that annual net sales for the Hershey Company were 5.1 billion dollars in 2008 The statement $f^{\prime}(8)=0.22$ tells us that in 2008 , annual net sales increase by about 0.22 billion dollars in the next year.
(b). $f(12)=5.98$
15. (a). The statement $f(10)=92.63$ tells us that meat production was 92.63 million metric tons in 2010. The statement $f^{\prime}(5)=0.64$ means that in 2010, US meat production increased by about 0.64 million metric tons in the next year.
(b). 98.83 million metric tons.
16. (a). The statement $f(20)=0.36$ means that 20 minutes after smoking a cigarette, there will be 0.36 mg of nicotine in the body. The statement $f^{\prime}(20)=-0.002$ means that 20 minutes after smoking a cigarette, about 0.002 mg of nicotine leaves the body in the next minute. The units are 20 minutes, 0.36 mg , and $-0.002 \mathrm{mg} /$ minute.
(b). 0.34 .
17. (a). The company hopes that increased advertising always brings in more customers instead of turning them away. There- fore, it hopes $f^{\prime}(a)$ is always positive.
(b). If $f^{\prime}(100)=2$, it means that if the advertising budget is $\$ 100,000$, an extra dollar spent on advertising will bring in about $\$ 2$ worth of sales. If $f^{\prime}(100)=0.5$, an extra dollar above $\$ 100$ thousand spent on advertising will bring in about $\$ 0.50$ worth of sales.
(c). If $f^{\prime}(100)=2$, then as we saw in part $(b)$, spending slightly more than $\$ 100,000$ will increase revenue by an amount greater than the additional expense, and thus more should be spent on advertising. If $f^{\prime}(100)=0.5$, then the increase in revenue is less than the additional expense, hence too much is being spent on advertising. The optimum amount to spend, $a$, is an amount that makes $f^{\prime}(a)=1$. At this point, the increases in advertising expenditures just pay for themselves. If $f^{\prime}(a)<1$, too much is being spent; if $f^{\prime}(a)>1$, more should be spent.
18. If $f^{\prime}(80,000)=2$, it means that if the budget for materials is $\$ 80,000$, another dollar spent on materials will bring in about $\$ 2$ more in revenue. If $f^{\prime}(80,000)=0.5$, another dollar spent on materials will bring in about $\$ 0.50$ more in revenue.

## Section $2.5-1,8,11,12,13$

1. 3 .
2. The slope of the revenue curve is greater than the slope of the cost curve at both $q_{1}$ and $q_{2}$, so the marginal revenue is greater at both production levels
3. (a) $\$ 4348$
(b) $\$ 11$
(c) The company loses $\$ 3$.
4. The marginal revenue is less than the marginal cost so the 90 th bus should not be added.
5. (a). 1.8 million dollars.
(b). $\$ 28,000$.
(c). $-\$ 35,000$
