Name:
Homework 5 solutions
Math 151, Applied Calculus, Spring 2018
Section 3.1 –10,11,17,23,26,28,43,60,63,65

10. $f'(x) = -4x^{-5}$.

11. $y' = 18x^2 + 8x - 2$.

17. $f'(z) = 6.1z^{-7.1}$.

23. $\frac{dz}{dt} = 2t$.

26. $y' = 15t^4 - \frac{5}{2}t^{-1/2} - \frac{7}{t^2}$.

28. $y' = 6t - \frac{6}{t^{3/2}} + \frac{2}{t^3}$.

43. $f(5) = 625cm$. $f'(t) = -6t$ so we have $f'(5) = -30cm/year$ In the year 2010, the sand dune will be 625 cm high and eroding at a rate of 30 centimeters per year.

60. $C'(q) = 4q$ so $C'(25)$ =100 per item This means that the cost of production increases by about \$100 when we add one unit to a production level of 25 units.

63. (a) $f(5) = 770$ bushels per acre. (b) $f'(5) = 40$ bushels per acre per pound of fertilizer. (c) More should be used, because at this level of use, more fertilizer will result in a higher yield. Fertilizers use should be increased until an additional unit results in a decrease in yield. i.e. until the derivative at that point becomes negative.

65. (a) $C(50) = \$14,750$. (b) $C'(50) = \$675$ per item. i.e At $q = 50$ costs will increase by about \$675 for one additional item of product produced.

Section 3.2 – 1,6,7,15,18,31,39,41,49,52

1. $\frac{dP}{dt} = 9t^2 + 2e^t$.

6. $\frac{dy}{dx} = \ln(2)2^x - 6x^{-4}$.

7. $\frac{dy}{dx} 4\ln(10)10^x - 3x^2$.

15. $\frac{dP}{dt} = 24e^{0.12t}$.

18. $P'(t) = 3000(\ln(1.02)(1.02)^t$.

31. $\frac{f'(t)}{f(t)} = -7$.

39. (a) $P(12) \approx 13,394$. There are 13,394 fish in the area after 12 months. (b) $P'(12) \approx 8037$ fish/month. The population is growing at a rate of approximately 8037 fish per month.

41. $f(2) = 6065$ If the product sells for \$2, then 6065 units can be sold. $f'(2) = -1516$. Thus, at a price of \$2, a \$1 increase in price results in a decrease in quantity sold of about 1516 units.

49. $C(50) \approx 1365$ $C'(50) \approx 18.27$. It costs about \$18.27 to produce an additional unit when the production level is 50 units.

52. The population in the US was growing faster because the rate of change of the US population in 2009 is 2.979 million people per year and for Mexico the rate of change is 1.247 million people per year.

Section 3.3 –2,8,13,18,24,35,36,50

2. $f'(x) = 99(1+x)^{98}$.

8. $y' = \dfrac{3s^2}{2\sqrt{s^3+1}}$.

13. $\dfrac{dy}{dt} = \dfrac{5}{5t+1}$.

18. $f'(x) = \dfrac{e^{-x}}{1-e^{-x}}$.

24. $f'(x) = \dfrac{e^x}{2\sqrt{e^x+1}}$.

35. (a) $\dfrac{dB}{dt} = P\left(1+\dfrac{r}{100}\right)^t \ln\left(1+\dfrac{r}{100}\right)$. This tells us how fast the amount of money in the bank is changing with respect to time for fixed initial investment P and interest rate r.
(b) $\dfrac{dB}{dr} = Pt\left(1+\dfrac{r}{100}\right)^{t-1} \dfrac{1}{100}$. This indicates how fast the amount of money changes with respect to the interest rate r , assuming fixed initial investment P and time t.

36. Marginal revenue is obtained by taking a derivative of the total revenue. At $q = 10$, Marginal revenue $= \$0.2/unit$.

50. (a) 16.89 dollars per hour (b) Thus, the difference is $34.01 - 20.14 = 13.87$ dollars per hour.
(c) 0.486 dollars per hour per year.

Section 3.4 – 5,9,13,19,23,29,36,38,40,42

5. $\dfrac{dy}{dt} = 2t(3t+1)^3 + 9t^2(3t+1)^2$.

9. $y' = (3t^2 - 14t)e^t + (t^3 - 7t^2 + 1)e^t$.

13. $f'(x) = 1 - \dfrac{3}{x^2}$.

19. $f'(w) = e^{w^2}(10w) + (5w^2+3)(e^{w^2})(2w)$.

23. $\dfrac{dz}{dt} = \dfrac{-2}{(1+t)^2}$.

29. $f'(x) = (ax)(e^{-bx}(-b)) + (a)e^{-bx}$.

36. $f(1) = 60.65mg, \quad f(5) = 41.04mg$
$f'(t) = 100e^{-0.5t} + 100t(e^{-0.5t}(-0.5)), \quad f'(1) = 30.33mg/hour, \quad f'(5) = -12.31mg/hour$ One hour after the drug was administered, the quantity of drug in the body is $60.65mg$ and the quantity is increasing at a rate of $30.33mgperhour$. Five hours after the drug was administered, the quantity in the body is $41.04mg$ and the quantity is decreasing at a rate of $12.31mgperhour$.

38. $q(10) \approx 2247, \quad q'(10) = -400e^{-0.8} \approx -180$ This means that at a price of \$10, a \$1 increase in price will result in a decrease in quantity demanded by 180 units.

40. (a) $R(p) = p \cdot 1000e^{-0.02p}$, (b) $R'(p) = 1000e^{-0.02p} + 1000pe^{-0.02p}$, (c) $R'(10) \approx 665$. a one dollar increase in price over \$10 will generate about \$655 in additional revenue.

42. (a) $f(140) = 15,000$ says that $15,000$ skateboards are sold when the cost is \$140 per board. $f'(140) = -100$ means that if the price is increased from \$140, roughly speaking, every dollar of increase will decrease the total sales by 100 boards
(b) $\frac{dR}{dp} = \frac{d}{dp}(p \cdot q) = \frac{d}{dp}(p \cdot f(p)) = f(p) + pf'(p)$ so $R'(140) = f(140) + 140f'(140) = 1000$. This means that the revenue will increase by about \$1000 if the price is raised by \$1.