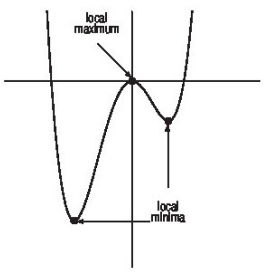
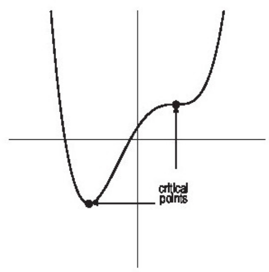
Name:
Homework 6 solutions
Math 151, Applied Calculus, Spring 2018
Section 4.1 – 1-4,5,20,23,24-27,38

1–4. For each graph look for the points where the slope of the tangent line is zero or $f'(x) = 0$.

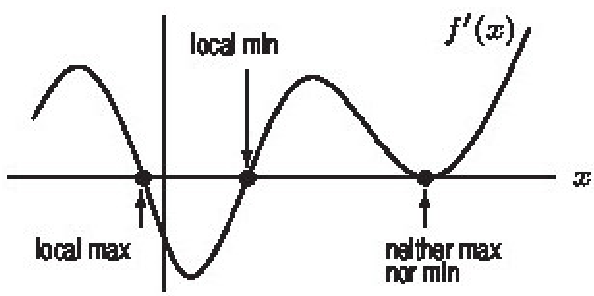
5 .





20. $f'(x) = 4x^3 - 12x^2 + 8$, notice that $f'(1) = 0$ then applying the second derivative test $f''(x) = 12x^2 - 24x$. Since $f''(1) = -12 < 0$, $x = 1$ is a local maximum.

23. The critical points of f are zeros of $f'(x)$. Just to the left of the first critical point $f'(x) > 0$, so f is increasing. Immediately to the right of the first critical point $f'(x) < 0$, so f is decreasing. Thus, the first point must be a maximum. To the left of the second critical point, $f'(x) < 0$, and to its right, $f'(x) > 0$; hence it is a minimum. On either side of the last critical point, $f'(x) > 0$, so it is neither a maximum nor a minimum.



24. (a) Increasing for $x > 0$, decreasing for $x < 0$.
    (b) $f(0)$ is a local and global minimum, and f has no global maximum

25. (a) Increasing for all x
    (b) No maxima or minima

26. (a) Decreasing for $x < 0$, increasing for $0 < x < 4$, and decreasing for $x > 4$.
    (b) $f(0)$ is a local minimum, and $f(4)$ is a local maximum.

27. (a) Decreasing for $x < -1$, increasing for $-1 < x < 0$, decreasing for $0 < x < 1$, and increasing for $x > 1$

    (b) $f(-1)$ and $f(1)$ are local minima, $f(0)$ is a local maximum

28. The function f has critical points at $x = 1$, $x = 3$, $x = 5$.

Section 4.2 – 1-4,12,13,15,24

1–4 We find inflection points by looking at the points on the graph where the graph changes concavity.

12 The graph has one critical point $x = \frac{5}{2}$ and no inflection points.

13 Critical points are $x = 1$ and $x = -1$ and the inflection point is at $x = 0$. Note you can use the online plotting calculator "Desmos" to check your work.

15. Critical points are $x = 1$ or $x = -2$ and the inflection point is $x = \dfrac{1}{2}$.

24. To find inflection points of the function f we must find points where $f''(x)$ changes sign. However, because $f''(x)$ is the derivative of $f'(x)$ any point where $f''(x)$ changes sign will be a local maximum or minimum on the graph of $f'(x)$

Section 4.3 – 1,3,8,17,18,38,39,42

1. The Global min occurs at the left end-point while the Global maximum is in the interior.

3. (a) IV, (b) I, (c) III, (d) II

8 True. If the maximum is not at an endpoint, then it must be at critical point of f. But $x = 0$ is the only critical point of $f(x) = x^2$ and it gives a minimum, not a maximum.

17. (a) $f'(x) = 6x^2 - 18x + 12$ and $f''(x) = 12x - 18$.

    (b) $x = 1, 2$ are critical points.

    (c) $x = \frac{3}{2}$ is an inflection point.

    (d) To find the global max/min we compare the values of the function at the end points and critical points. Global max is 10 at $x = 3$ and the global min is -7.5 at $x = -0.5$.

    (e) Check your plot using the online graphing calculator Desmos.

18 (a) $f'(x) = 3x^2 - 6x - 9$, $f''(x) = 6x - 6$.

    (b) $x = -1, 3$ are critical points.

    (c) $x = 1$ is an inflection point.

    (d) Global max at $x = -1$ and $x = 4$ and the global min at $x = -5$.

    (e) Check your plot using the online graphing calculator Desmos.

38 (a) Since the crow makes $n(x)$ trips to a height of x metres, the total vertical distance upward is $h(x) = x \cdot n(x) = x + \frac{27}{x}$ metres.

    (b) At the min $h'(x) = 0$ You should find that the critical point is $x = 5.2$ metres. You can check using the second derivative that is indeed a min.

39 For simplicity, we can rewrite the function using the properties of the ln so that

$$I(S) = 192(\ln(S) - ln(762) - A + 763$$

so that

$$I'(S) = \frac{192}{S} - 1$$

The critical point is $S = 192$, use the second derivative to check that this is a maximum. The maximum possible number of infected children is therfore $I(192) = 306$.

42 (a) $q(0) = 0$ so there is initially non of the drug in the bloodstream.

(b) The maximum value of $q(t)$ occurs when $q'(t) = 0$

$$\begin{aligned} q'(t) = 20(-e^{-t} + 2e^{-2t}) &= 0 \\ -e^{-t} &= -2e^{-2t} \\ \frac{e^{-t}}{e^{-2t}} &= 2 \\ e^{t} &= 2 \\ t &= \ln(2) \end{aligned}$$

(c) In the long run we expect the amount of drug to leave the body. Mathemtically, this is because for large values of t, $e^{-t} \longrightarrow 0$ and $e^{-2t} \longrightarrow 0$

Section 4.4 – 1,2,3,4,6,7,15-23

1. The profit function is positive when $R(q) > C(q)$, and negative when $C(q) > R(q)$. Its positive for $5.5 < q < 12.5$, and negative for $0 < q < 5.5$ and $12.5 < q$. Profit is maximized when $R(q) > C(q)$ and $R'(q) = C'(q)$ which occurs about $q = 9.5$.

2. $q = 245$ items.

3. Profit is maximized at $q = 75$ units. Maximum profit is \$6865.

4. See notes from class.

6. (a). The value of $C(0)$ represents the fixed costs before production, that is, the cost of producing zero units, incurred for initial investments in equipment, and so on.

   (b). The marginal cost decreases slowly, and then increases as quantity produced increases.

   (c). Concave down implies decreasing marginal cost, while concave up implies increasing marginal cost.

   (d). An inflection point of the cost function is (locally) the point of maximum or minimum marginal cost.

   (e). One would think that the more of an item you produce, the less it would cost to produce extra items. In economic terms, one would expect the marginal cost of production to decrease, so we would expect the cost curve to be concave down. In practice, though, it eventually becomes more expensive to produce more items, because workers and resources may become scarce as you increase production. Hence after a certain point, the marginal cost may rise again. This happens in oil production, for example.

7. (a) \$9, , (b) $-$\$3, Profit is maximized at $q = 78$ so we must have $C'(78) = R'(78)$.

15 Since marginal revenue is larger than marginal cost around $q = 2000$, as you produce more of the product your revenue increases faster than your costs, so profit goes up, and maximal profit will occur at a production level above 2000

16 (a). The marginal cost at q = 400 is the slope of the tangent line to C(q) at q = 400. Looking at the graph, we can estimate a slope of about 1. Thus, the marginal cost is about \$1.

(b). At q = 500, we can see that slope of the cost function is greater than the slope of the revenue function. Thus, the marginal cost is greater than the marginal revenue and thus the 500th item will incur a loss. So, the company should not produce the 500th item.

(c). The quantity which maximizes profit is at the point where marginal cost equals marginal revenue. This occurs when the slope of $R(q)$ equals $C(q)$, which occurs at approximately $q = 400$. Thus, the company should produce about 400 items.

17. When 10 items are produced, each additional item produced gives approximately \$0.20 in additional revenue.

18. $R(q) = (45 - 0.01q)q$, The revenue is maximized at $q = 2250$. The total revenue is $\$50,625$.

19. (a) If $q = 3000$, the demand equation gives $p = 700.02 \cdot 3000 = 10$. That is, at a price of \$10, 3000 people attend. At this price, $Revenue = 3000people \cdot 10dollars/person = \$30,000$.To find total revenue at a price of \$20, first find the attendance at this price. Substituting $p = 20$ into the demand equation, $p = 700.02q$, gives $20 = 700.02q$. Solving for q, we get $q = 2500$. That is, at a price of \$20, attendance is 2500 people, and $Revenue = 2500 \cdot 20 = \$50,000$. Notice that, although demand is reduced, the revenue is higher at a price of \$20 than at \$10.

(b) $R(q) = 70q - 0.02q^2$.

(c) $q = 1750$.

(d) $q = 35$.

(e) The maximum revenue is $61,250$.

20. $p = 4.5, q = 3600$.

21. $p = 14$.

22 (a) $\pi = -5q^2 + 3994q - 5$, (b) $q = 399.4$, (c) $\pi(399.4) = \$797,596.80$.

23 (a) $C(q) = 10,000 + 2q$. (b) $q = 37,820 - 5544p$, (c) $\pi = -0.000018q^2 + 4.822q - 10,0000$. (d) $\pi = 22,294dollars$.

Section 4.5 – 1-4,6,8,9,10

1. (a) Since the graph is concave down, the average cost gets smaller as q increases. This is because the cost per item gets smaller as q increases. There is no value of q for which the average cost is minimized since for any q_0 larger than q the average cost at q_0 is less than the average cost at q. Graphically, the average cost at q is the slope of the line going through the origin and through the point $(q, C(q))$. As q gets larger, the average cost decreases.

   (b) The average cost will be minimized at some q for which the line through $(0, 0)$ and $(q, c(q))$ is tangent to the cost curve.

2. (a) \$1.60 per unit, (b) $a(q) = \frac{C(q)}{q}$, (c) $q \approx 18,000$

3. (a)(i) \$8 per unit. (a)(ii) \$4 per unit. (b) $q \approx 30$.

4. (a) \$12 (b) \$37 and \$14.50.

8. (a) Monthly profit is \$21,600
   (b) Since additional units produced cost about \$3 each, which is above the average cost, producing them increases average cost. Since additional pairs of slippers cost about \$3 to produce and can be sold for \$20, you can increase your profit by making and selling them. This is a case where marginal revenue, which is \$20 per slipper, is greater than marginal cost, which is \$3 per pair of slippers.
   (c) You should recommend increase in production, since that increases profit. The fact that average cost of production increases is irrelevant to your decision.

9 (a) $C(q) = 0.01q^3 - 0.6q^2 + 13q$.
   (b) $MC(q) = 0.03q^2 - 1.2q + 13$.
   (c) The minimum average cost is $a(30) = 4$ dollars per item.
   (d) The marginal cost at $q = 30$ must equal the minimum average cost $a(30) = 4$

10 (a) The marginal cost tells us that additional units produced would cost about \$10 each, which is below the average cost, so producing them would reduce average cost.
   (b) It is impossible to tell, one needs to factor in costs.

Section 4.6 –1,9,10,11,13,17

1. (a) The quantity demanded decreases by about $0.5(3\%) = 1.5\%$
   (b) The quantity demanded increases by about $0.5(3\%) = 1.5\%$

9. $E = \frac{2}{3}$.

10. $E = 2.5$

11 Use the formula stated in class // (a) $E = 0.470$, (b) At $p = 1.25$, $E = 0.943$, At $p = 1.5E = 0.970$. e.t.c

13. See in class worksheet solutions.

17 $E < 1$ so increasing price increases revenue.