MA 490: Homework 3: Laplace's Equation and Fourier Series (due Wednesday, 21 March)

Reading : Sections 2.5, 3.1–3.3

1. Use the method of separation of variables to solve

$$\begin{aligned} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} &= 0, \quad 0 \le x \le L, 0 \le y \le H \\ \frac{\partial u}{\partial x}(0,y) &= 0 \\ \frac{\partial u}{\partial x}(L,y) &= 0 \\ u(x,0) &= 0 \\ u(x,H) &= g(x) \end{aligned}$$

   Hint: In class we solved Laplace's equation with 3 homogeneous Dirichlet boundary conditions and 1 non-homogeneous boundary condition. You have a similar set up here with the difference being that some of your homogenous boundary conditions are Neumann.

2. Solve Laplace's equation *outside* a circular disk ($r \ge a$) subject to the boundary condition

$$u(a,\theta) = 5\cos(4\theta)$$

   Hint: In class we solved Laplace's equation *inside* the disk. Use polar coordinates and seperation of variables and assume that the solution remains bounded outside the disk i.e $|u(r,\theta)| < \infty$ as $r \longrightarrow \infty$.

3. Problem 3.3.1b on page 110.

4. Problem 3.3.2a on page 110.

5. Find the Fourier *cosine* series of the function $|sin(x)|$ on the interval $[-\pi, \pi]$. Use your Fouier series to show that

$$\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} = \frac{1}{2}$$