Reading: Sections 2.5, 3.1–3.3

1. Use the method of separation of variables to solve

\[
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 \leq x \leq L, 0 \leq y \leq H
\]

\[
\frac{\partial u}{\partial x}(0, y) = 0
\]

\[
\frac{\partial u}{\partial x}(L, y) = 0
\]

\[
u(x, 0) = 0
\]

\[
u(x, H) = g(x)
\]

Hint: In class we solved Laplace’s equation with 3 homogeneous Dirichlet boundary conditions and 1 non-homogeneous boundary condition. You have a similar set up here with the difference being that some of your homogenous boundary conditions are Neumann.

2. Solve Laplace’s equation outside a circular disk \((r \geq a)\) subject to the boundary condition

\[
u(a, \theta) = 5 \cos(4\theta)
\]

Hint: In class we solved Laplace’s equation inside the disk. Use polar coordinates and separation of variables and assume that the solution remains bounded outside the disk i.e \(|u(r, \theta)| < \infty\) as \(r \to \infty\).

3. Problem 3.3.1b on page 110.

4. Problem 3.3.2a on page 110.

5. Find the Fourier cosine series of the function \(\left|\sin(x)\right|\) on the interval \([-\pi, \pi]\). Use your Fourier series to show that

\[
\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} = \frac{1}{2}
\]