

MA 490: Homework 4: Term by term differentiation & integration of Fourier series (due Wednesday April 4)

Reading : Sections 3.4, 3.5, 4.1-4.2 and 4.4.

1. Suppose $f(x)$ and $f'(x)$ are piecewise smooth. Prove that the Fourier cosine series of a continuous $f(x)$ can be differentiated term by term.
2. Problem 3.4.6 on page 121.
3. Solve the following non-homogeneous problem

$$\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2} + \sin(3\pi x)$$

subject to

$$u(0, t) = 0, \quad u(1, t) = 0, \quad u(x, 0) = \sin(\pi x)$$

4. Problem 3.5.2 on page 126.
5. Starting with the Fourier cosine series of x ,

$$x = \frac{L}{2} - \frac{4L}{\pi^2} \left(\cos \frac{\pi x}{L} + \frac{\cos 3\pi x/L}{3^2} + \frac{\cos 5\pi x/L}{5^2} + \dots \right), \quad 0 \leq x \leq L$$

prove that

$$1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \dots = \frac{\pi^3}{32}$$

using term by term integration of the series and evaluating the resulting series at an appropriate point.