Reading: Sections 3.4, 3.5, 4.1-4.2 and 4.4.

1. Suppose \( f(x) \) and \( f'(x) \) are piecewise smooth. Prove that the Fourier cosine series of a continuous \( f(x) \) can be differentiated term by term.

2. Problem 3.4.6 on page 121.

3. Solve the following non-homogeneous problem

\[
\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2} + \sin(3\pi x)
\]

subject to

\[
u(0, t) = 0, \quad u(1, t) = 0, \quad u(x, 0) = \sin(\pi x)
\]

4. Problem 3.5.2 on page 126.

5. Starting with the Fourier cosine series of \( x \),

\[
x = \frac{L}{2} - \frac{4L}{\pi^2} \left( \cos \frac{\pi x}{L} + \frac{\cos 3\pi x/L}{3^2} + \frac{\cos 5\pi x/L}{5^2} + \cdots \right), \quad 0 \leq x \leq L
\]

 prove that

\[
1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots = \frac{\pi^3}{32}
\]

using term by term integration of the series and evaluating the resulting series at an appropriate point.