MA 490: Homework 5 (due Monday April 30)

Reading: Sections: 5.1-5.5; 8.1-8.3

1. Assuming that $\rho, \alpha, \beta$ are functions of $x$ with $\beta = cp$ where $c$ is a constant. Show that the spatial problem corresponding to

$$\rho \frac{\partial^2 u}{\partial t^2} = T_0 \frac{\partial^2 u}{\partial x^2} + \alpha u + \beta \frac{\partial u}{\partial t}$$

is a Sturm-Liouville differential equation.

2. Problem 5.3.3 on page 161.

3. Show that $\lambda > 0$ for the eigenvalue problem

$$\frac{d^2 \phi}{dx^2} + (\lambda - x^2)\phi = 0$$

with

$$\frac{d\phi}{dx}(0) = 0, \frac{d\phi}{dx}(1) = 0$$

Is $\lambda = 0$ an eigenvalue? Explain.

4. Consider

$$c\rho \frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left( K_0 \frac{\partial u}{\partial x} \right) + \alpha u,$$

where $c, \rho, K_0,$ and $\alpha$ are functions of $x$ subject to

$$u(0, t) = 0$$

$$u(L, t) = 0$$

$$u(x, 0) = f(x)$$

By showing that the spatial problem is of Sturm-Liouville form and therefore the eigenfunctions are known:

(a) Solve the initial value problem.

(b) Briefly discuss $\lim_{t \to \infty} u(x, t)$.

5. Problems 5.5.1d, g on page 174.

6. Problem 5.6.1a, b on page 188.

7. Consider the eigenvalue problem

$$\frac{d^2 \phi}{dx^2} + \lambda \phi = 0$$

subject to $\frac{d\phi}{dx}(0) = 0$ and $\frac{d\phi}{dx}(L) + h\phi(L) = 0$ with $h > 0$.

(a) Use the Rayleigh quotient to prove that $\lambda > 0$

(b) Determine all the eigenvalues graphically. Estimate the large eigenvalues.

8. Problem 8.3.6 on page 353.