MA 490: Homework 5 (due Monday April 30)

Reading : Sections: 5.1-5.5; 8.1-8.3

1. Assuming that ρ, α, β are functions of x with $\beta = c\rho$ where c is a constant. Show that the spatial problem corresponding to
$$\rho \frac{\partial^2 u}{\partial t^2} = T_0 \frac{\partial^2 u}{\partial x^2} + \alpha u + \beta \frac{\partial u}{\partial t}$$
is a Sturm-Liouville differential equation.

2. Problem 5.3.3 on page 161.

3. Show that $\lambda > 0$ for the eigenvalue problem
$$\frac{d^2\phi}{dx^2} + (\lambda - x^2)\phi = 0$$
with
$$\frac{d\phi}{dx}(0) = 0, \frac{d\phi}{dx}(1) = 0$$
Is $\lambda = 0$ an eigenvalue? Explain.

4. Consider
$$c\rho \frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(K_0 \frac{\partial u}{\partial x}\right) + \alpha u,$$
where c, ρ, K_0, and α are functions of x subject to
$$\begin{aligned} u(0,t) &= 0 \\ u(L,t) &= 0 \\ u(x,0) &= f(x) \end{aligned}$$
By showing that the spatial problem is of Sturm-Liouville form and therefore the eigenfunctions are known:
   (a) Solve the initial value problem.
   (b) Briefly discuss $\lim_{t\to\infty} u(x,t)$.

5. Problems 5.5.1d, g on page 174.

6. Problem 5.6.1a, b on page 188.

7. Consider the eigenvalue problem
$$\frac{d^2\phi}{dx^2} + \lambda\phi = 0$$
subject to $\frac{d\phi}{dx}(0) = 0$ and $\frac{d\phi}{dx}(L) + h\phi(L) = 0$ with $h > 0$.
   (a) Use the Rayleigh quotient to prove that $\lambda > 0$
   (b) Determine all the eigenvalues graphically. Estimate the large eigenvalues.

8. Problem 8.3.6 on page 353.