$\frac{d}{dx}\left(p(x)\frac{du}{dx}\right) + q(x)\phi + \lambda\sigma\phi = 0$

Theorem

1. *All the eigenvalues λ are real.*
2. *There exists an infinite number of eigenvalues $\lambda_1 < \lambda_2 < \ldots < \lambda_n < \lambda_{n+1} < \ldots$.*
3. *Corresponding to each eigenvalue λ_n, there is an eigenfunction, $\phi_n(x)$ (unique to within an arbitrary multiplicative constant). $\phi_n(x)$ has exactly $n-1$ zeros for $a < x < b$.*
4. *The eigenfunctions $\phi_n(x)$ form a "complete" set, meaning that any piecewise smooth function, $f(x) \sim \sum_{n=1}^{\infty} a_n\phi_n(x)$.*
5. *The eigenfunctions are orthogonal relative to a weight function $\sigma(x)$, i.e* $\int_a^b \phi_n(x)\phi_m(x)\sigma(x)\,dx = 0 \quad \textit{if } \lambda_n \neq \lambda_m.$
6. *Any eigenvalue can be related to its eigenfunction by the* **Rayleigh quotient:**

$$\lambda = \frac{-p\phi\frac{d\phi}{dx}\big|_a^b + \int_a^b \left[p(\frac{d\phi}{dx})^2 - q\phi^2\right] dx}{\int_a^b \phi^2\sigma\, dx}.$$