Name:
Homework 2 solutions
Math 151, Applied Calculus, Spring 2018

Section 1.5 – 1,2,4,8,10,12,16,28,29,30,32

1. a). Town (i) has the largest percent growth rate, at 12%.
   b). Town (ii) has the largest initial population, at 1000.
   c). Yes, town (iv) is decreasing in size, since the decay factor is 0.9, which is less than 1.

2. In each of these cases recall thet if
$$A = A_0(a)^t$$
then the initial amount is A_0 and if the growth factor $a > 1$ we have exponential growth otherwise we have decay.

4. (a) II (b) I (c) III (d) IV

8. This problem is similar to our in-class example. You want to find an equation of the form $P(x) = P_0a^x$. The points $(0, 3)$ and $(2, 12)$ are on the graph so
$$3 = y_0a^0 = y_0 \quad \text{and } 12 = y_0a^2 = 3 \cdot a^2$$
You should get $a = 2$ and $y_0 = 3$ so the equation is $y = 3 \cdot (2^x)$.

10. a). $G = 685.4(1.02)^t$
    b). $G = 685.4 + 7t$

12. (a) $80 - 4t$ (b) $80(0.95)^t$

16. $CPI - 234(1.018)^t$

28. From the statements provided we have $P_0a^3 = 1600$ and $P_0a^1 = 1000$ We have 2 equations and 2 unknows we solve for a and P_0 as done in class: From the first equation
$$P_0 = \frac{1600}{a^3}$$
plugging that into the second equation
$$\frac{1600}{a^2} = 1000 \Longrightarrow 1.6 = a^2$$
so $a = 1.6^{1/2} \approx 1.265$. Now plug in this value of a and solve for P_0 as
$$P_0 = \frac{1600}{\sqrt{1.6}} = 790.569$$
so we conclude (a) $a = 1.265, P_0 = 790.569$ (b) The initial quantity is growing at a rate of 26.5%.

29. The population is assumed to be exponential so $P(t) = P_0e^{kt}$ assuming t is the time since 1998 so $P_0 = 5.937$, so $P(t) = 5.937e^{kt}$, now we need to solve for k, we can use the other point provided
$$P(16) = 7.238 \Longrightarrow 5.937e^{16k} = 7.238$$

to sove for k

$$\begin{aligned}
e^{16t} &= \frac{7.238}{5.937} && (1)\\
ln(e^{16t}) &= ln(\frac{7.238}{5.937}) && (2)\\
16k &= ln(\frac{7.238}{5.937}) && (3)\\
& && (4)
\end{aligned}$$

solving for k, we get 0.0124 therefore our population is $P(t) = 5.937e^{0.0124t}$ billion. For part (b) Notice that in 2015, $t = 17$ so compute $P(17) = 7.33bilion$

30. This problem was solved in class on Friday. 1.7%. Here we are looking for annual percentage increase therefore we want to find a function of the form $P = P_0a^t$ such that $P(0) = 190,205$ and $P(5) = 174,989$. You should get $P(t) = 190,205(0.932)^t$ corresponding to a annual decrease of 1.7%.

32. (a) Since the assumption is that the mussels are growing linerly we need to find the equation of a line given the 2 points $(0, 2700)$ and $(1, 3186)$ you should get $y = 2700 + 486t$. For (b) we need to find an exponential function of the form $y = P_0a^t$ to obtain the annual percentage increase. When $t = 0, y = P_0 = 2700$, to solve for a use the other point as

$$3186 = 2700a^1$$

to get $a = 1.18$ yeilding an 18% annual growth rate.

Section 1.6 – 2,18,21-24,33,35,36,40,41

2. $t = 1.209$

18. $t \approx 0.9163$

21. Initial quantity is 5 and growth rate is 7%

22. Initial quantity is 7.7 and growth rate is -8%

23. Initial quantity is 15 and growth rate is -6% (continuous decay)

24. Initial quantity is 3.2 and growth rate is 3% (continuous)

33. Using the provided information, we can create 2 equations $P_0e^{3k} = 140$ and $P_0e^{1k} = 100$. Using the first equation we can solve for P_0 and plug into the second equation

$$P_0 = \frac{140}{e^{3k}}$$

plug that into second equation

$$\begin{aligned}
\left(\frac{140}{e^{3k}}\right)e^{1k} &= 100\\
\frac{140}{e^{2k}} &= 100\\
\frac{140}{100} &= e^{2k}\\
\ln\left(\frac{140}{100}\right) &= \ln\left(e^{2k}\right) = 2k
\end{aligned}$$

so that $k = 0.168$. now we can solve for $P_0 = \frac{140}{e^{3(0.168)}} = 84.575$ so we have (a) $a = 0.168$ and $P_0 = 84.575$ (b) P_0 is the initial amount fowing at a continuous rate of 16.8%.

35. (a) Continous percentage growth rate is 6% $k = -0.916, P_0 = 1560.684$ (b) Initial quantity is 1560.684 decaying at a continuous rate of 91.6%.

36. (a) annual decay rate is 12% (b) $P = 25e^{-0.128t}$.

40. $f(t) = 84e^{-0.091t}$, $f(17) = 18.882$ million barrels per day

41. (a) (i) $P = 1000(1.05)^t$ (ii) $P = 1000e^{0.05t}$ (b) (i) 1629 (ii) 1649