Figure 1.4: Increasing and decreasing functions

Problems for Section 1.1

1. Which graph in Figure 1.5 best matches each of the following stories?[2] Write a story for the remaining graph.

(a) I had just left home when I realized I had forgotten my books, so I went back to pick them up.
(b) Things went fine until I had a flat tire.
(c) I started out calmly but sped up when I realized I was going to be late.

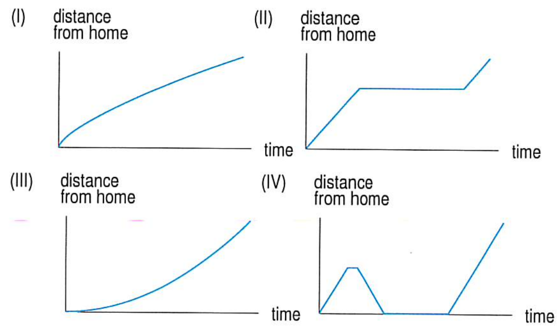


Figure 1.5

In Problems 2–5, use the description of the function to sketch a possible graph. Put a label on each axis and state whether the function is increasing or decreasing.

2. The height of a sand dune is a function of time, and the wind erodes away the sand dune over time.

3. The amount of carbon dioxide in the atmosphere is a function of time, and is going up over time.

4. The number of air conditioning units sold is a function of temperature, and goes up as the temperature goes up.

5. The noise level, in decibels, is a function of distance from the source of the noise, and the noise level goes down as the distance increases.

6. The population of Washington DC grew from 1900 to 1950, stayed approximately constant during the 1950s, and decreased from about 1960 to 2005. Graph the population as a function of years since 1900.

7. Let $W = f(t)$ represent wheat production in Argentina,[3] in millions of metric tons, where t is years since 2006. Interpret the statement $f(4) = 14$ in terms of wheat production.

8. The concentration of carbon dioxide, $C = f(t)$, in the atmosphere, in parts per million (ppm), is a function of years, t, since 1960.

(a) Interpret $f(40) = 370$ in terms of carbon dioxide.[4]
(b) What is the meaning of $f(50)$?

9. **(a)** The graph of $r = f(p)$ is in Figure 1.6. What is the value of r when p is 0? When p is 3?
(b) What is $f(2)$?

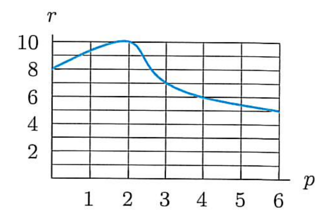


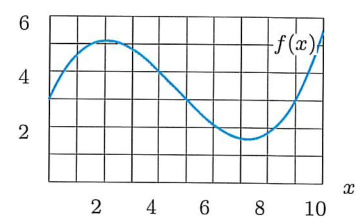
Figure 1.6

For the functions in Problems 10–14, find $f(5)$.

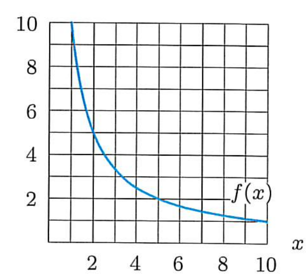
10. $f(x) = 2x + 3$

11. $f(x) = 10x - x^2$

12.



13.



14.

x	1	2	3	4	5	6	7	8
$f(x)$	2.3	2.8	3.2	3.7	4.1	5.0	5.6	6.2

[2] Adapted from Jan Terwel, "Real Math in Cooperative Groups in Secondary Education." *Cooperative Learning in Mathematics*, ed. Neal Davidson, p. 234 (Reading: Addison Wesley, 1990).
[3] http://ageconsearch.umn.edu/bitstream/115558/2/AAE680.pdf, accessed September 2012.
[4] *Vital Signs 2007-2008*, The Worldwatch Institute, W.W. Norton & Company, 2007, p. 43.

15. Let $y = f(x) = x^2 + 2$.

(a) Find the value of y when x is zero.
(b) What is $f(3)$?
(c) What values of x give y a value of 11?
(d) Are there any values of x that give y a value of 1?

In Problems 16–19 the function $S = f(t)$ gives the average annual sea level, S, in meters, in Aberdeen, Scotland,[5] as a function of t, the number of years before 2008. Write a mathematical expression that represents the given statement.

16. In 1983 the average annual sea level in Aberdeen was 7.019 meters.

17. The average annual sea level in Aberdeen in 2008.

18. The average annual sea level in Aberdeen was the same in 1865 and 1911.

19. The average annual sea level in Aberdeen increased by 1 millimeter from 2007 to 2008.

20. **(a)** A potato is put in an oven to bake at time $t = 0$. Which of the graphs in Figure 1.7 could represent the potato's temperature as a function of time?
(b) What does the vertical intercept represent in terms of the potato's temperature?

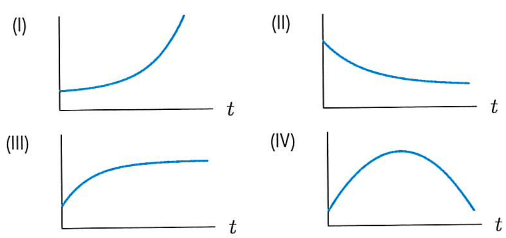


Figure 1.7

21. An object is put outside on a cold day at time $t = 0$. Its temperature, $H = f(t)$, in °C, is graphed in Figure 1.8.

(a) What does the statement $f(30) = 10$ mean in terms of temperature? Include units for 30 and for 10 in your answer.
(b) Explain what the vertical intercept, a, and the horizontal intercept, b, represent in terms of temperature of the object and time outside.

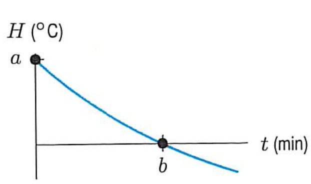


Figure 1.8

22. In the Andes mountains in Peru, the number, N, of species of bats is a function of the elevation, h, in feet above sea level, so $N = f(h)$.

(a) Interpret the statement $f(500) = 100$ in terms of bat species.
(b) What are the meanings of the vertical intercept, k, and horizontal intercept, c, in Figure 1.9?

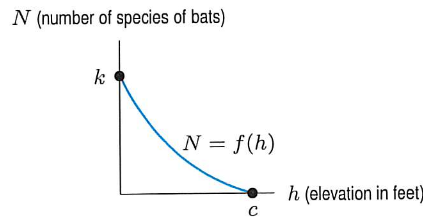


Figure 1.9

23. In tide pools on the New England coast, snails eat algae. Describe what Figure 1.10 tells you about the effect of snails on the diversity of algae.[6] Does the graph support the statement that diversity peaks at intermediate predation levels?

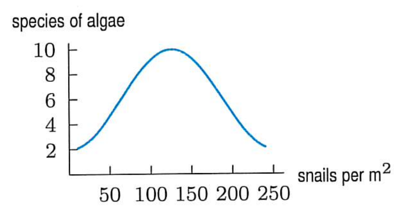


Figure 1.10

24. Figure 1.11 shows the amount of nicotine, $N = f(t)$, in mg, in a person's bloodstream as a function of the time, t, in hours, since the person finished smoking a cigarette.

(a) Estimate $f(3)$ and interpret it in terms of nicotine.
(b) About how many hours have passed before the nicotine level is down to 0.1 mg?
(c) What is the vertical intercept? What does it represent in terms of nicotine?
(d) If this function had a horizontal intercept, what would it represent?

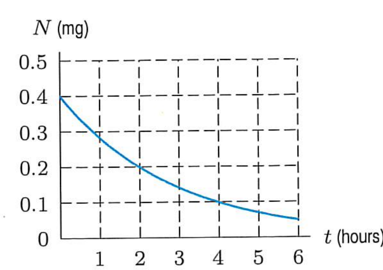


Figure 1.11

[5] www.decc.gov.uk, accessed June 2011
[6] Rosenzweig, M.L., *Species Diversity in Space and Time,* p. 343 (Cambridge: Cambridge University Press, 1995).

25. A deposit is made into an interest-bearing account. Figure 1.12 shows the balance, B, in the account t years later.

(a) What was the original deposit?
(b) Estimate $f(10)$ and interpret it.
(c) When does the balance reach \$5000?

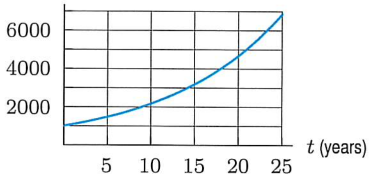


Figure 1.12

26. The use of CFCs (chlorofluorocarbons) has declined since the 1987 Montreal Protocol came into force to reduce the use of substances that deplete the ozone layer. World annual CFC consumption, $C = f(t)$, in million tons, is a function of time, t, in years since 1987. (CFCs are measured by the weight of ozone that they could destroy.)

(a) Interpret $f(10) = 0.2$ in terms of CFCs.[7]
(b) Interpret the vertical intercept of the graph of this function in terms of CFCs.
(c) Interpret the horizontal intercept of the graph of this function in terms of CFCs.

27. When a patient with a rapid heart rate takes a drug, the heart rate plunges dramatically and then slowly rises again as the drug wears off. Sketch the heart rate against time from the moment the drug is administered.

28. The gas mileage of a car (in miles per gallon) is highest when the car is going about 45 miles per hour and is lower when the car is going faster or slower than 45 mph. Graph gas mileage as a function of speed of the car.

29. After an injection, the concentration of a drug in a patient's body increases rapidly to a peak and then slowly decreases. Graph the concentration of the drug in the body as a function of the time since the injection was given. Assume that the patient has none of the drug in the body before the injection. Label the peak concentration and the time it takes to reach that concentration.

30. Financial investors know that, in general, the higher the expected rate of return on an investment, the higher the corresponding risk.

(a) Graph this relationship, showing expected return as a function of risk.
(b) On the figure from part (a), mark a point with high expected return and low risk. (Investors hope to find such opportunities.)

31. The number of sales per month, S, is a function of the amount, a (in dollars), spent on advertising that month, so $S = f(a)$.

(a) Interpret the statement $f(1000) = 3500$.
(b) Which of the graphs in Figure 1.13 is more likely to represent this function?
(c) What does the vertical intercept of the graph of this function represent, in terms of sales and advertising?

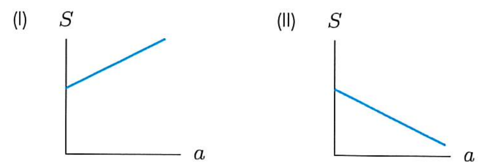


Figure 1.13

32. Figure 1.14 shows fifty years of fertilizer use in the US, India, and the former Soviet Union.[8]

(a) Estimate fertilizer use in 1970 in the US, India, and the former Soviet Union.
(b) Write a sentence for each of the three graphs describing how fertilizer use has changed in each region over this 50-year period.

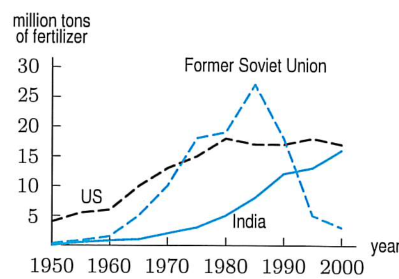


Figure 1.14

33. The six graphs in Figure 1.15 show frequently observed patterns of age-specific cancer incidence rates, in number of cases per 1000 people, as a function of age.[9] The scales on the vertical axes are equal.

(a) For each of the six graphs, write a sentence explaining the effect of age on the cancer rate.
(b) Which graph shows a relatively high incidence rate for children? Suggest a type of cancer that behaves this way.
(c) Which graph shows a brief decrease in the incidence rate at around age 50? Suggest a type of cancer that might behave this way.

---

[7] *Vital Signs 2007-2008*, The Worldwatch Institute, W.W. Norton & Company, 2007, p. 47.
[8] The Worldwatch Institute, *Vital Signs 2001*, p. 32 (New York: W.W. Norton, 2001).
[9] Abraham M. Lilienfeld, *Foundations of Epidemiology*, p. 155 (New York: Oxford University Press, 1976).

(d) Which graph or graphs might represent a cancer that is caused by toxins which build up in the body over time? (For example, lung cancer.) Explain.

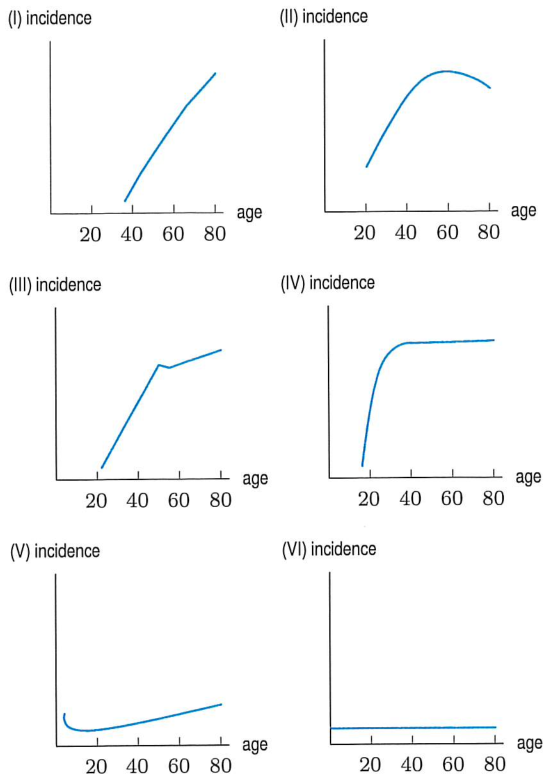


Figure 1.15

34. Table 1.2 shows the average annual sea level, S, in meters, in Aberdeen, Scotland,[10] as a function of time, t, measured in years before 2008.

Table 1.2

t	0	25	50	75	100	125
S	7.094	7.019	6.992	6.965	6.938	6.957

(a) What was the average sea level in Aberdeen in 2008?

(b) In what year was the average sea level 7.019 meters? 6.957 meters?

(c) Table 1.3 gives the average sea level, S, in Aberdeen as a function of the year, x. Complete the missing values.

Table 1.3

x	1883	?	1933	1958	1983	2008
S	?	6.938	?	6.992	?	?

Problems 35–38 ask you to plot graphs based on the following story: "As I drove down the highway this morning, at first traffic was fast and uncongested, then it crept nearly bumper-to-bumper until we passed an accident, after which traffic flow went back to normal until I exited."

35. Driving speed against time on the highway

36. Distance driven against time on the highway

37. Distance from my exit vs time on the highway

38. Distance between cars vs distance driven on the highway

1.2 LINEAR FUNCTIONS

Probably the most commonly used functions are the *linear functions*, whose graphs are straight lines. The chirp-rate and the Honda depreciation functions in the previous section are both linear. We now look at more examples of linear functions.

Olympic and World Records

During the early years of the Olympics, the height of the men's winning pole vault increased approximately 8 inches every four years. Table 1.4 shows that the height started at 130 inches in 1900, and increased by the equivalent of 2 inches a year between 1900 and 1912. So the height was a linear function of time.

Table 1.4 *Winning height (approximate) for Men's Olympic pole vault*

Year	1900	1904	1908	1912
Height (inches)	130	138	146	154

If y is the winning height in inches and t is the number of years since 1900, we can write

$$y = f(t) = 130 + 2t.$$

[10] www.decc.gov.uk, accessed June 2011.