$$\begin{aligned} q &= b + mp \\ 90 &= b + (-2)(10) \\ 90 &= b - 20 \\ 110 &= b. \end{aligned}$$

Thus, the equation of the line is

$$q = 110 - 2p.$$

(b) If we now consider p as a linear function of q, then p is the dependent variable and q is the independent variable. We have

$$\text{Slope} = m = \frac{\Delta p}{\Delta q} = \frac{10-5}{90-100} = \frac{5}{-10} = -0.5.$$

The units of the slope are dollars per ton.

Since p is a linear function of q, we have $p = b + mq$ and $m = -0.5$. To find b, we substitute any point from the table, such as $p = 10$, $q = 90$, into this equation:

$$\begin{aligned} p &= b + mq \\ 10 &= b + (-0.5)(90) \\ 10 &= b - 45 \\ 55 &= b. \end{aligned}$$

Thus, the equation of the line is

$$p = 55 - 0.5q.$$

Alternatively, we could take our answer to part (a), that is $q = 110 - 2p$, and solve for p.

Appendix A shows how to fit a linear function to data that is not exactly linear.

Families of Linear Functions

Formulas such as $f(x) = b + mx$, in which the constants m and b can take on various values, represent a *family of functions*. All the functions in a family share certain properties—in this case, the graphs are lines. The constants m and b are called *parameters*. Figures 1.19 and 1.20 show graphs with several values of m and b. Notice the greater the magnitude of m, the steeper the line.

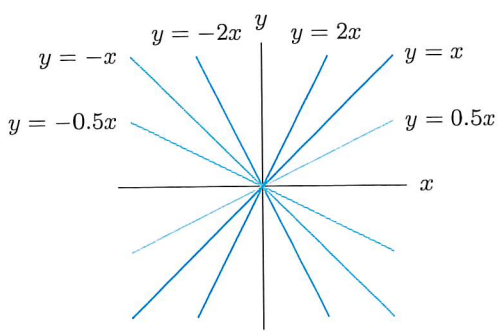

Figure 1.19: The family $y = mx$ (with $b = 0$)

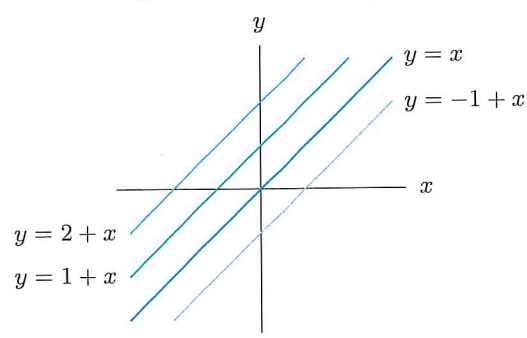

Figure 1.20: The family $y = b + x$ (with $m = 1$)

Problems for Section 1.2

For Problems 1–4, find an equation for the line that passes through the given points.

1. $(0, 2)$ and $(2, 3)$

2. $(0, 0)$ and $(1, 1)$

3. $(-2, 1)$ and $(2, 3)$

4. $(4, 5)$ and $(2, -1)$

For Problems 5–8, determine the slope and the y-intercept of the line whose equation is given.

5. $7y + 12x - 2 = 0$

6. $3x + 2y = 8$

7. $12x = 6y + 4$

s **8.** $-4y + 2x + 8 = 0$

9. Figure 1.21 shows four lines given by equation $y = b + mx$. Match the lines to the conditions on the parameters m and b.

(a) $m > 0, b > 0$ **(b)** $m < 0, b > 0$
(c) $m > 0, b < 0$ **(d)** $m < 0, b < 0$

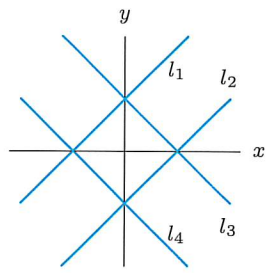

Figure 1.21

10. **(a)** Which two lines in Figure 1.22 have the same slope? Of these two lines, which has the larger y-intercept?
(b) Which two lines have the same y-intercept? Of these two lines, which has the larger slope?

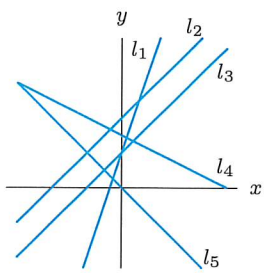

Figure 1.22

11. A city's population was 30,700 in the year 2010 and is growing by 850 people a year.

(a) Give a formula for the city's population, P, as a function of the number of years, t, since 2010.
(b) What is the population predicted to be in 2020?
(c) When is the population expected to reach 45,000?

12. A company rents cars at \$40 a day and 15 cents a mile. Its competitor's cars are \$50 a day and 10 cents a mile.

(a) For each company, give a formula for the cost of renting a car for a day as a function of the distance traveled.
(b) On the same axes, graph both functions.
(c) How should you decide which company is cheaper?

13. Figure 1.23 shows the distance from home, in miles, of a person on a 5-hour trip.

(a) Estimate the vertical intercept. Give units and interpret it in terms of distance from home.
(b) Estimate the slope of this linear function. Give units, and interpret it in terms of distance from home.
(c) Give a formula for distance, D, from home as a function of time, t in hours.

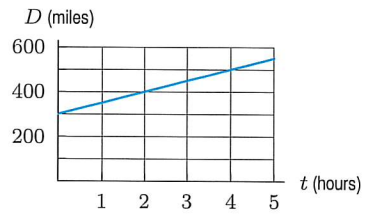

Figure 1.23

14. Which of the following tables could represent linear functions?

(a)

x	0	1	2	3
y	27	25	23	21

(b)

t	15	20	25	30
s	62	72	82	92

(c)

u	1	2	3	4
w	5	10	18	28

15. For each table in Problem 14 that could represent a linear function, find a formula for that function.

16. A cell phone company charges a monthly fee of \$25 plus \$0.05 per minute. Find a formula for the monthly charge, C, in dollars, as a function of the number of minutes, m, the phone is used during the month.

17. Annual revenue R from McDonald's restaurants worldwide can be estimated by $R = 19.1 + 1.8t$, where R is in billion dollars and t is in years since January 1, 2005.[13]

(a) What is the slope of this function? Include units. Interpret the slope in terms of McDonald's revenue.
(b) What is the vertical intercept of this function? Include units. Interpret the vertical intercept in terms of McDonald's revenue.
(c) What annual revenue does the function predict for 2015?
(d) When is annual revenue predicted to hit 35 billion dollars?

[13]Based on McDonald's Annual Report 2007, accessed at www.mcdonalds.com, May 2012.

18. A company's pricing schedule in Table 1.5 is designed to encourage large orders. (A gross is 12 dozen.) Find a formula for:

(a) q as a linear function of p.
(b) p as a linear function of q.

Table 1.5

q (order size, gross)	3	4	5	6
p (price/dozen)	15	12	9	6

19. World milk production rose at an approximately constant rate between 2000 and 2012.[14] See Figure 1.24.

(a) Estimate the vertical intercept and interpret it in terms of milk production.
(b) Estimate the slope and interpret it in terms of milk production.
(c) Give an approximate formula for milk production, M, as a function of t.

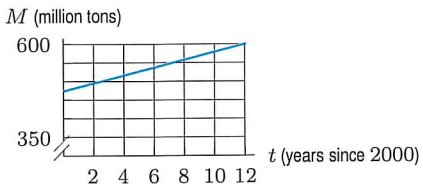

Figure 1.24

20. The percentage of people, P, below the poverty level in the US[15] is given in Table 1.6.

(a) Find a formula for the percentage in poverty as a linear function of time in years since 2000.
(b) Use the formula to predict the percentage in poverty in 2006.
(c) What is the difference between the prediction and the actual percentage, 12.3%?

Table 1.6

Year (since 2000)	0	1	2	3
P (percentage)	11.3	11.7	12.1	12.5

21. World grain production was 1241 million tons in 1975 and 2048 million tons in 2005, and has been increasing at an approximately constant rate.[16]

(a) Find a linear function for world grain production, P, in million tons, as a function of t, the number of years since 1975.
(b) Using units, interpret the slope in terms of grain production.
(c) Using units, interpret the vertical intercept in terms of grain production.
(d) According to the linear model, what is the predicted world grain production in 2015?
(e) According to the linear model, when is grain production predicted to reach 2500 million tons?

22. Annual sales of music compact discs (CDs) have declined since 2000. Sales were 942.5 million in 2000 and 384.7 million in 2008.[17]

(a) Find a formula for annual sales, S, in millions of music CDs, as a linear function of the number of years, t, since 2000.
(b) Give units for and interpret the slope and the vertical intercept of this function.
(c) Use the formula to predict music CD sales in 2012.

23. Search and rescue teams work to find lost hikers. Members of the search team separate and walk parallel to one another through the area to be searched. Table 1.7 shows the percent, P, of lost individuals found for various separation distances, d, of the searchers.[18]

Table 1.7

Separation distance d (ft)	20	40	60	80	100
Approximate percent found, P	90	80	70	60	50

(a) Explain how you know that the percent found, P, could be a linear function of separation distance, d.
(b) Find P as a linear function of d.
(c) What is the slope of the function? Give units and interpret the answer.
(d) What are the vertical and horizontal intercepts of the function? Give units and interpret the answers.

24. In a California town, the monthly charge for waste collection is \$8 for 32 gallons of waste and \$12.32 for 68 gallons of waste.

(a) Find a linear formula for the cost, C, of waste collection as a function of the number of gallons of waste, w.
(b) What is the slope of the line found in part (a)? Give units and interpret your answer in terms of the cost of waste collection.
(c) What is the vertical intercept of the line found in part (a)? Give units and interpret your answer in terms of the cost of waste collection.

[14] http://www.dairyco.org.uk/library/market-information/datum/world-milk-production.aspx, accessed May 2012.
[15] www.census.gov/hhes/www/poverty/histpov/hstpov2.html
[16] *Vital Signs 2007-2008*, The Worldwatch Institute, W.W. Norton & Company, 2007, p. 21.
[17] *The World Almanac and Book of Facts 2008* (New York).
[18] From *An Experimental Analysis of Grid Sweep Searching*, by J. Wartes (Explorer Search and Rescue, Western Region, 1974).

25. The number of species of coastal dune plants in Australia decreases as the latitude, in °S, increases. There are 34 species at 11°S and 26 species at 44°S.[19]

(a) Find a formula for the number, N, of species of coastal dune plants in Australia as a linear function of the latitude, l, in °S.

(b) Give units for and interpret the slope and the vertical intercept of this function.

(c) Graph this function between $l = 11°$S and $l = 44°$S. (Australia lies entirely within these latitudes.)

26. Table 1.8 gives the average weight, w, in pounds, of American men in their sixties for height, h, in inches.[20]

(a) How do you know that the data in this table could represent a linear function?

(b) Find weight, w, as a linear function of height, h. What is the slope of the line? What are the units for the slope?

(c) Find height, h, as a linear function of weight, w. What is the slope of the line? What are the units for the slope?

Table 1.8

h (inches)	68	69	70	71	72	73	74	75
w (pounds)	166	171	176	181	186	191	196	201

Problems 27–32 concern the maximum heart rate (MHR), which is the maximum number of times a person's heart can safely beat in one minute. If MHR is in beats per minute and a is age in years, the formulas used to estimate MHR, are

$$\text{For females: MHR} = 226 - a,$$

$$\text{For males: MHR} = 220 - a.$$

27. Which of the following is the correct statement?

(a) As you age, your maximum heart rate decreases by one beat per year.

(b) As you age, your maximum heart rate decreases by one beat per minute.

(c) As you age, your maximum heart rate decreases by one beat per minute per year.

28. Which of the following is the correct statement for a male and female of the same age?

(a) Their maximum heart rates are the same.

(b) The male's maximum heart rate exceeds the female's.

(c) The female's maximum heart rate exceeds the male's.

29. What can be said about the ages of a male and a female with the same maximum heart rate?

30. Recently[21] it has been suggested that a more accurate predictor of MHR for both males and females is given by

$$\text{MHR} = 208 - 0.7a.$$

(a) At what age do the old and new formulas give the same MHR for females? For males?

(b) Which of the following is true?

(i) The new formula predicts a higher MHR for young people and a lower MHR for older people than the old formula.

(ii) The new formula predicts a lower MHR for young people and a higher MHR for older people than the old formula.

(c) When testing for heart disease, doctors ask patients to walk on a treadmill while the speed and incline are gradually increased until their heart rates reach 85 percent of the MHR. For a 65-year-old male, what is the difference in beats per minute between the heart rate reached if the old formula is used and the heart rate reached if the new formula is used?

31. Experiments[22] suggest that the female MHR decreases by 12 beats per minute by age 21, and by 19 beats per minute by age 33. Is this consistent with MHR being approximately linear with age?

32. Experiments[23] suggest that the male MHR decreases by 9 beats per minute by age 21, and by 26 beats per minute by age 33. Is this consistent with MHR being approximately linear with age?

33. Let y be the percent increase in annual US national production during a year when the unemployment rate changes by u percent. (For example, $u = 2$ if unemployment increases from 4% to 6%.) Okun's law states that

$$y = 3.5 - 2u.$$

(a) What is the meaning of the number 3.5 in Okun's law?

(b) What is the effect on national production of a year when unemployment rises from 5% to 8%?

(c) What change in the unemployment rate corresponds to a year when production is the same as the year before?

(d) What is the meaning of the coefficient -2 in Okun's law?

[19] Rosenzweig, M.L., *Species Diversity in Space and Time,* p. 292 (Cambridge: Cambridge University Press, 1995).

[20] Adapted from "Average Weight of Americans by Height and Age," *The World Almanac* (New Jersey: Funk and Wagnalls, 1992), p. 956.

[21] www.physsportsmed.com/issues/2001/07_01/jul01news.htm, accessed January 4, 2005.

[22] www.css.edu/users/tboone2/asep/May2002JEPonline.html, accessed January 4, 2005.

[23] www.css.edu/users/tboone2/asep/May2002JEPonline.html, accessed January 4, 2005.

34. An Australian[24] study found that, if other factors are constant (education, experience, etc.), taller people receive higher wages for the same work. The study reported a "height premium" for men of 3% of the hourly wage for a 10 cm increase in height; for women the height premium reported was 2%. We assume that hourly wages are a linear function of height, with slope given by the height premium at the average hourly wage for that gender.

(a) The average hourly wage[25] for a 178 cm Australian man is AU\$29.40. Express the average hourly wage of an Australian man as a function of his height, x cm.

(b) The average hourly wage for a 164 cm Australian woman is AU\$24.78. Express the average hourly wage of an Australian woman as a function of her height, y cm.

(c) What is the difference in average hourly wages between men and women of height 178 cm?

(d) Is there a height for which men and women are predicted to have the same wage? If so, what is it?

1.3 AVERAGE RATE OF CHANGE AND RELATIVE CHANGE

Average Rate of Change

In the previous section, we saw that the height of the winning Olympic pole vault increased at an approximately constant rate of 2 inches/year between 1900 and 1912. Similarly, the world record for the mile decreased at an approximately constant rate of 0.4 seconds/year. We now see how to calculate rates of change when they are not constant.

Example 1 Table 1.9 shows the height of the winning pole vault at the Olympics[26] during the 1960s and 1990s. Find the rate of change of the winning height between 1960 and 1968, and between 1992 and 2000. In which of these two periods did the height increase faster than during the period 1900–1912?

Table 1.9 *Winning height in men's Olympic pole vault (approximate)*

Year	1960	1964	1968	···	1992	1996	2000
Height (inches)	185	201	213	···	228	233	232

Solution From 1900 to 1912, the height increased by 2 inches/year. To compare the 1960s and 1990s, we calculate

$$\text{Average rate of change of height 1960 to 1968} = \frac{\text{Change in height}}{\text{Change in time}} = \frac{213-185}{1968-1960} = 3.5 \text{ inches/year.}$$

$$\text{Average rate of change of height 1992 to 2000} = \frac{\text{Change in height}}{\text{Change in time}} = \frac{232-228}{2000-1992} = 0.5 \text{ inches/year.}$$

Thus, on average, the height was increasing more quickly during the 1960s than from 1900 to 1912. During the 1990s, the height was increasing more slowly than from 1900 to 1912.

In Example 1, the function does not have a constant rate of change (it is not linear). However, we can compute an *average rate of change* over any interval. The word average is used because the rate of change may vary within the interval. We have the following general formula.

If y is a function of t, so $y = f(t)$, then

$$\textbf{Average rate of change} \text{ of } y \text{ between } t=a \text{ and } t=b = \frac{\Delta y}{\Delta t} = \frac{f(b)-f(a)}{b-a}.$$

The units of average rate of change of a function are units of y per unit of t.

[24] "Study finds tall people at top of wages ladder", Yahoo News, May 17, 2009.
[25] Australian Fair Pay Commission, August 2007.
[26] *The World Almanac and Book of Facts, 2005*, p. 866 (New York).