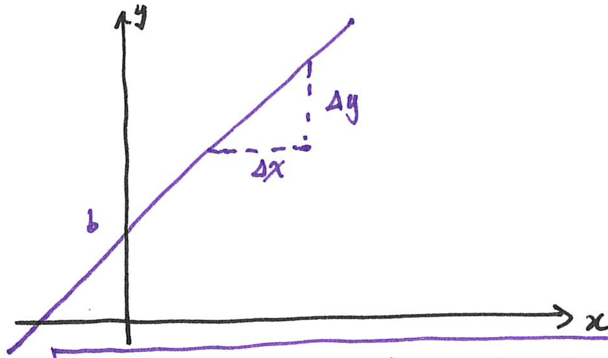
1.2 Linear Functions

Functions whose graphs are straight lines



Slope $= \dfrac{\text{change in } y}{\text{change in } x} = \dfrac{\Delta y}{\Delta x} = m$

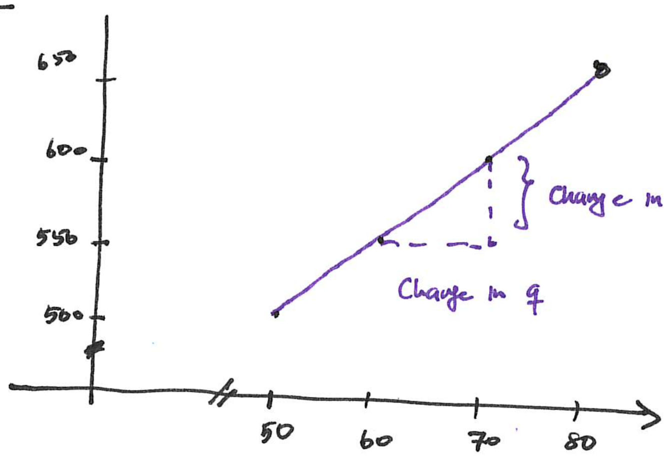
$y = mx + b$, where b is the y intercept.

$\boxed{\text{* The slope, } m \text{ is the rate of change of } y \text{ with respect to } x.}$

Back to our cost function

q	50	60	70	80	90
$C(q)$	500	550	600	650	700

Graph



$C(q)$ is a linear function

Slope $= \dfrac{\Delta C}{\Delta q} = \dfrac{600-550}{70-60}$

$= \dfrac{50}{10}$

$= \$5/\text{item}.$

The slope in this case is the cost of producing one additional item. (constant RATE OF INCREASE)

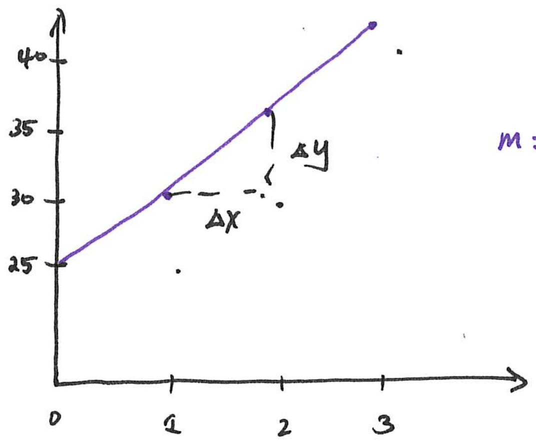
General linear functions ($y = mx + b$), where m = slope and b - y intercept.

Suppose

x	0	1	2	3
$f(x)$	25	30	35	40

From the table, how can we guess that this is a linear function?

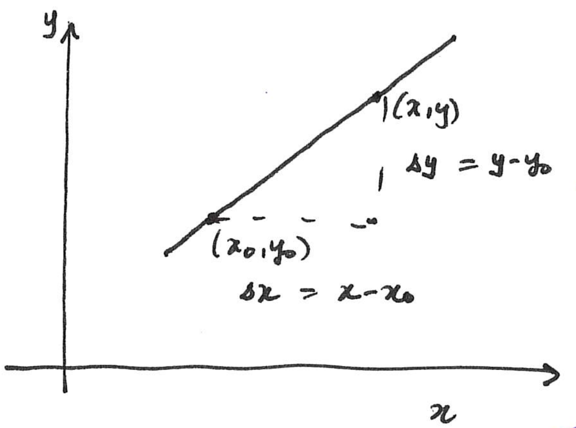
Graph



$$m = \frac{\Delta y}{\Delta x} = \frac{35-30}{2-1} = 5$$

The formula is $y = 5x + 25$.

Equation of a line given 2 points & slope (point-slope form).



$$\boxed{m = \frac{y - y_0}{x - x_0}}$$

Solving for y

$$m(x - x_0) = y - y_0 \Rightarrow \boxed{y - y_0 = m(x - x_0)}$$

Example

Equation of line with slope 2 passing through $(1,2)$

$$y - 2 = 2(x-1)$$

$$y = 2 + 2x - 2$$

$$y = 2x.$$

Equation of a line given 2 points

Equation of line passing through (0,2) and (2,3)

$$m = \frac{\Delta y}{\Delta x} = \frac{3-2}{2-0} = \frac{1}{2}$$

We can then use the point-slope form

$$y - y_0 = m(x - x_0)$$

$$\boxed{y - 3 = \tfrac{1}{2}(x-2)} \quad \text{or} \quad y = 3 + \frac{x}{2} - 1$$

$$\boxed{y = \frac{x}{2} + 2}$$

Determining slope and y-intercept from equation

Given $7y + 12x - 2 = 0$

$$\boxed{\text{Convert into the form } y = mx+b}$$

$$7y + 12x - 2 = 0$$

$$7y = 2 - 12x$$

$$y = \frac{2}{7} - \frac{12}{7}x \quad \Rightarrow \quad \text{intercept} = \frac{2}{7}$$

$$\text{Slope} = -\frac{12}{7}$$

Example

A city's population was 30,700 in the year 2010 and is growing by 850 people per year.

(a) Give a formula for the city's population, P as a function of t, the number of years since 2010.

(b) What is the predicted population in 2020?

(c) When is the population expected to reach 45,000?

(a) $P(t) = 30{,}700 + 850t$

(b) Year 2010 is $t=0$, Year 2020, $t=10$.

$P(10) = 30{,}700 + 850 \cdot 10 = 39{,}200.$

(c) ~~Fin.~~
t such that

$$45{,}000 = 30{,}700 + 850t$$

$$45{,}000 - 30{,}700 = 850t$$

$$\Rightarrow \quad t = \frac{14{,}300}{850} \quad \Rightarrow \quad t = 16.82$$

so in the year 2026.

Example (They do) Type out Section 1.2

The annual revenue for McDonalds can be estimated by

$$R(t) = 19.1 + 1.8t$$

when R is the revenue in billions and
t is time since 2005.

(a) What is the slope of R (include units) (interpret in terms of Revenue)

slope $= 1.8$ billion dollars/year.

The revenue is increasing at a rate of 1.8 billion dollars/year.

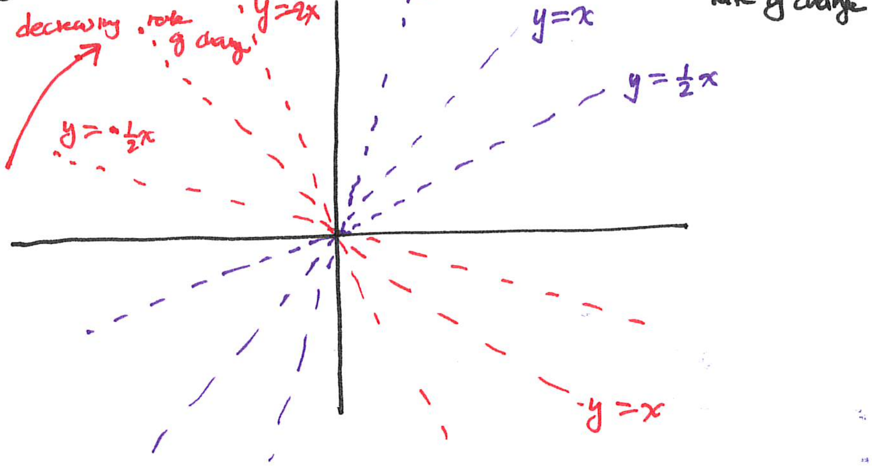
(b) What is the vertical intercept and interpret.

19.1 billion dollars.

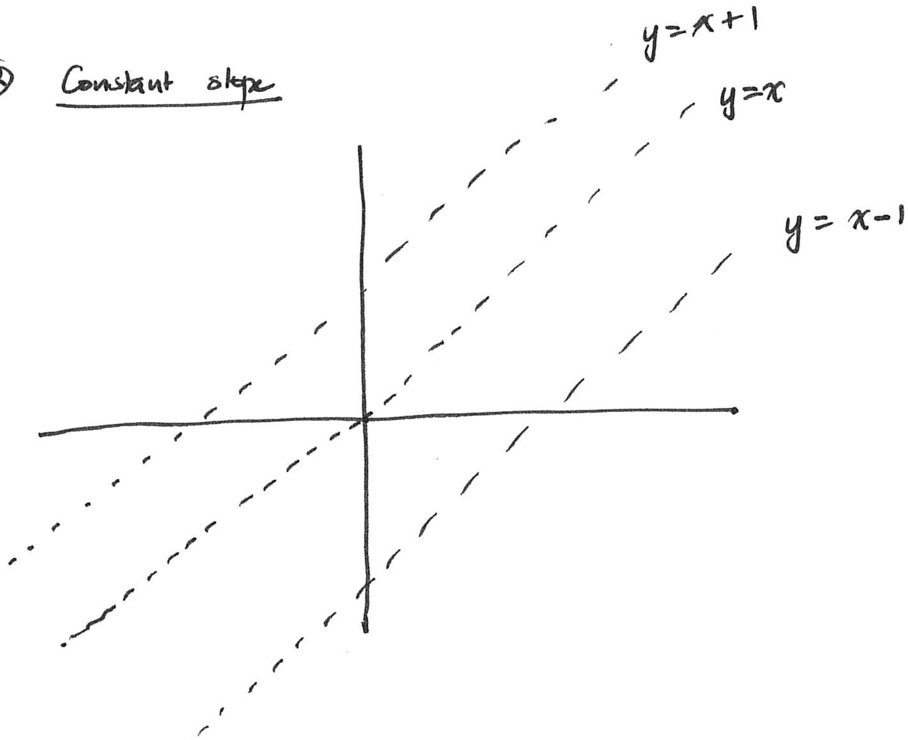
The revenue in 2005 was \$19.1 billion.

Families of Linear functions

① lines with 0 intercept



② Constant slope



Summary.

Linear functions describe a CONSTANT rate of change.

Revenue is increasing at \$50 ~~year~~ per year.

$y = \underline{\underline{m}}x + b.$