Section 1.3

Average Rate of change and Relative change

(1) Average Rate of Change

What if the rate of change is not constant?

eg Attendence at NFL games in millions of fans

Year	2003	2004	2005	2006	2007
Attendence	21.64	21.71	21.79	22.20	22.26

Notice that the rate of change is not constant

2003 − 2004 ⇒ 21.71 − 21.64 = 0.07 mil/yr

2004 − 2005 ⇒ 21.79 − 21.71 = 0.08 mil/yr

2005 − 2006 ⇒ 22.20 − 21.79 = 0.41 mil/yr.

2006 − 2007 ↳ 0.06 mil/yr.

Average rate of change in attendence from 2003 to 2007

$$= \frac{22.26 - 21.64}{2007 - 2003} = 0.155 \text{ mil/year} \quad (155\,000 \text{ fans/year}).$$

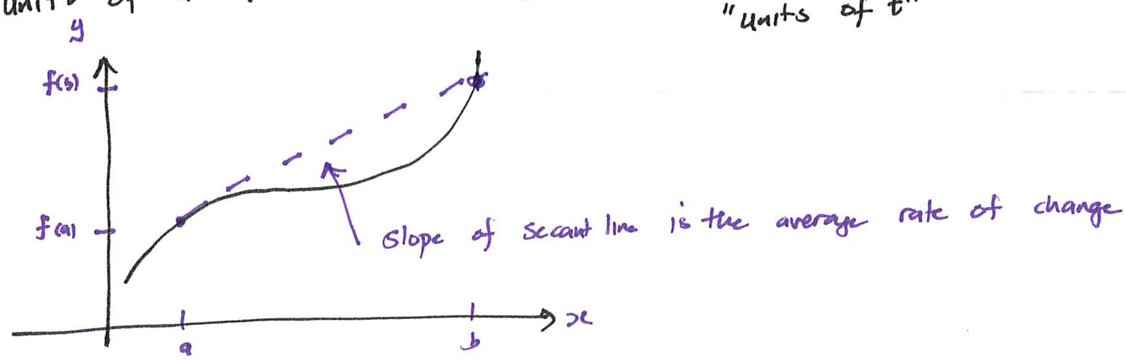
Notice that if we take the average of the rates of change

$$\frac{0.07 + 0.08 + 0.41 + 0.06}{4} = 0.155 \text{ mil/year}$$

In general for any function $y = f(t)$ defined on an interval $[a,b]$, the average rate of change of y between $t = a$ and $t = b$ is

$$\frac{\Delta y}{\Delta t} = \frac{f(b) - f(a)}{b - a}$$

Units of average rate of change are $\dfrac{\text{"units of } y\text{"}}{\text{"units of } t\text{"}}$

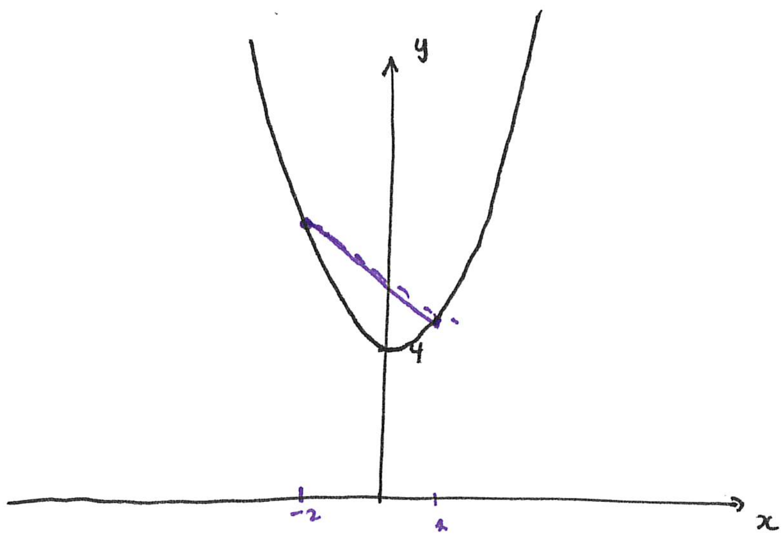


Example #2

$f(x) = 3x^2 + 4$, find the average rate of change between $x = -2$ and $x = 1$

$$\text{Average rate of change} = \frac{f(1) - f(-2)}{1 - (-2)} = \frac{(3(1)^2 + 4) - (3(-2)^2 + 4)}{3}$$

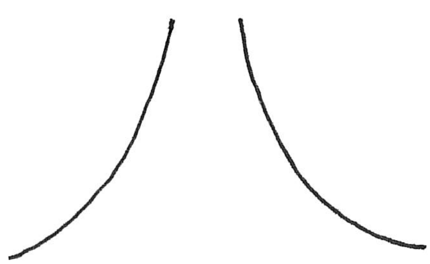
$$= -\frac{11}{3}$$



VISUALIZING RATE OF CHANGE - Shapes of graphs.

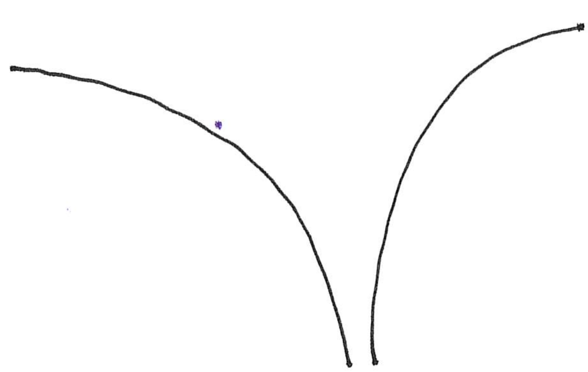
CONCAVITY

1. A graph of a function is concave up if the graph bends upwards as we move from left to right.



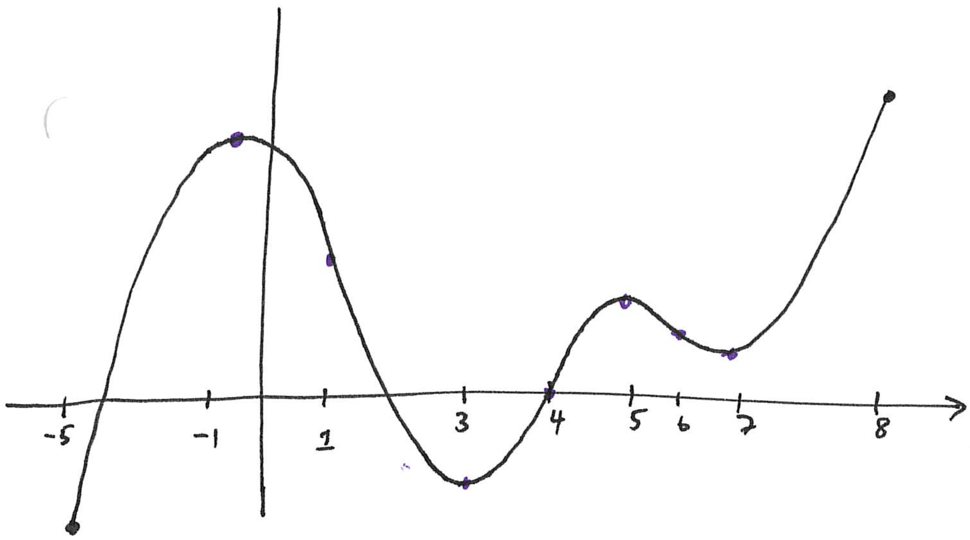
concave up

2. A graph is concave down if it bends downwards as we move from left to right



concave down.

Example #3



(i) Increasing and concave up [3,4] and [7,8]

(ii) Increasing and concave down [-5,-1] and [4,5]

(iii) Decreasing and concave up [1,3] and [6,7]

(iv) Decreasing and concave down [-1,1] and [5,6].

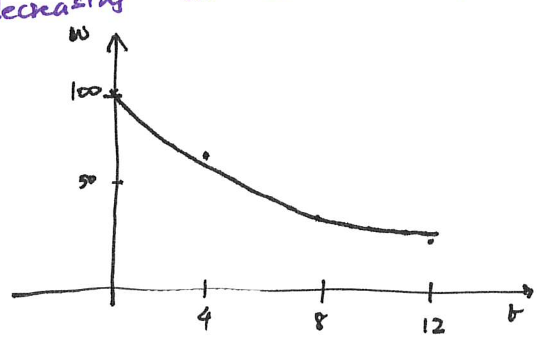
Example #4

Given a function in table format $w = f(t)$

⇒ Is the function increasing / decreasing. Concave up or down?

t	0	4	8	12	16	20	24
w	100	58	32	24	20	18	17

The function is decreasing and concave up because the values of w are decreasing at a decreasing rate.



Relative Rate of change

QUESTION

Is a population increase of 1000 a significant change?

ANSWER

It depends on the size of the community.

e.g For a small town of 2000 people

$$\text{Relative change} = \frac{\text{Change in population}}{\text{Initial population}} = \frac{1000}{2000} = 0.5$$

This represents a 50% increase!

However, for a large city of 10 million people

$$\text{Relative change} = \frac{1000}{10\,000\,000} = 0,0001$$

a 0.01% increase in population.

Relative change

Suppose a quantity changes from P_0 to P_1, then

The relative change in a quantity $P = \dfrac{\text{Change in } P}{P_0} = \dfrac{P_1 - P_0}{P_0}$.

Note:

1. The relative change has no units
2. May be expressed as a percentage.

VELOCITY, DISTANCE AND SPEED.

Suppose at time t, a particle's position is given by $s(t)$

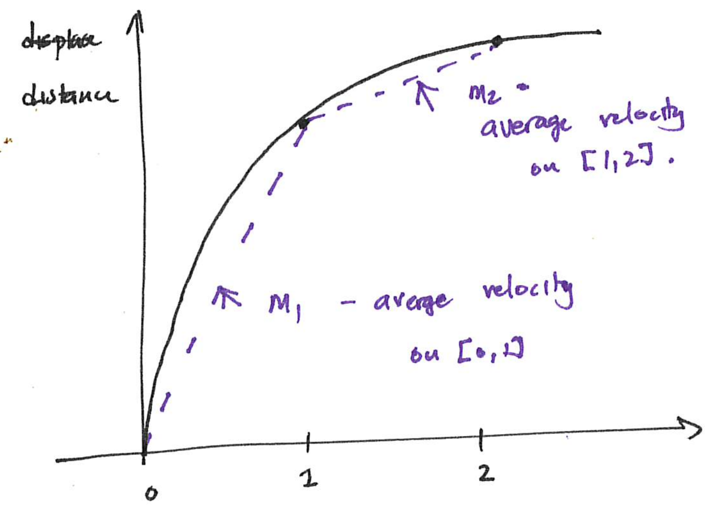
t (s)	0	3	6	10	13
$s(t)$ (m)	0	72	92	144	180

Find the average velocity of the particle from $t=3$ to $t=10$.

Average velocity is the average rate of change of distance w.r.t time.

i.e $\dfrac{\text{change in distance}}{\text{change in time}}$

Average velocity on $[3,10]$ $= \dfrac{144-72}{10-3} = \dfrac{72}{7} = 10.28\,m/s.$



$m_1 > m_2$ so the average velocity is greater on $[0,1]$ compared to $[1,2]$.

Ratio of relative changes

When the price is \$1.00, a store sells 3,000 items, when the price goes up to \$1.25, the quantity sold drops to 2,700.

Relative change in price of items $= \dfrac{1.25 - 1.00}{1.00} = 0.25.$ (25%)

Relative change in quantity sold $= \dfrac{2700 - 3000}{3000} = \dfrac{-300}{3000} = -0.1$ (10%)

We can compute the ratio (in absolute terms)

$$\left| \frac{\text{Relative change in quantity}}{\text{Relative change in price}} \right| = \left| \frac{-0.1}{0.25} \right| = \frac{10}{25} = \underline{0.4.}$$

so the number of items sold decreases by 0.4% when the price ~~drops~~ increases by 1%.

This ratio is called the <u>ELASTICITY</u> (measures how much changes in price affects quantity sold).

They do

#25

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