Average Rate of change and Relative change

(i) AVERAGE RATE OF CHANGE

What if the rate of change is not constant?

Attendance at NFL games in millions of fans

<table>
<thead>
<tr>
<th>Year</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attendance</td>
<td>21.64</td>
<td>21.71</td>
<td>21.79</td>
<td>22.20</td>
<td>22.26</td>
</tr>
</tbody>
</table>

Notice that the rate of change is not constant

\[ \frac{2003-2004}{2004-2005} = \frac{21.71-21.64}{21.79-21.71} = 0.8\text{ mil/yr} \]

\[ \frac{2005-2006}{2006-2007} = \frac{22.20-21.79}{0.41\text{ mil/yr}} = 0.06\text{ mil/yr} \]

Average rate of change in attendance from 2003 to 2007

\[ \frac{22.26 - 21.64}{2007 - 2003} = \frac{0.155}{4} = 0.155 \text{ mil/year (155,000 fans/year)} \]

Notice that if we take the average of the rates of change

\[ \frac{0.07 + 0.08 + 0.41 + 0.06}{4} = 0.155 \text{ mil/year} \]

In general, for any function \( f(t) \) defined on an interval \([a, b]\), the average rate of change of \( y \) between \( t = a \) and \( t = b \) is

\[ \Delta y \over \Delta t = \frac{f(b) - f(a)}{b-a} \]

Units of average rate of change are \"units of \( y \)\" / \"units of \( t \)\"
Example #2

\[ f(x) = 3x^2 + 4 \text{, find the average rate of change between } x = -2 \text{ and } x = 1 \]

\[
\text{Average rate of change} = \frac{f(1) - f(-2)}{1 - (-2)} = \frac{3(1)^2 + 4}{3} - \frac{3(-2)^2 + 4}{2} \\
= -\frac{11}{3}
\]

Visualizing Rate of Change - Shapes of graphs.

Concavity

1. A graph of a function is concave up if the graph bends upwards as we move from left to right.

2. A graph is concave down if it bends downwards as we move from left to right.
Example #3

(i) Increasing and concave up \([3,4]\) and \([7,8]\)

(ii) Increasing and concave down \([-5,-1]\) and \([4,5]\)

(iii) Decreasing and concave up \([1,2]\) and \([6,7]\)

(iv) Decreasing and concave down \([-1,1]\) and \([5,6]\).

Example #4

Given a function in table format \(W = f(t)\)

<table>
<thead>
<tr>
<th>(t)</th>
<th>0</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>(W)</td>
<td>100</td>
<td>58</td>
<td>32</td>
<td>24</td>
<td>20</td>
<td>16</td>
<td>17</td>
</tr>
</tbody>
</table>

\(\Rightarrow\) Is the function increasing/decreasing, concave up or down?

The function is decreasing and concave up because the values of \(W\) are decreasing at a decreasing rate.
**Question**

Is a population increase of 1000 a significant change?

**Answer**

It depends on the size of the community.

E.g., for a small town of 2000 people

\[
\text{Relative change} = \frac{\text{Change in population}}{\text{Initial population}} = \frac{1000}{2000} = 0.5
\]

This represents a 50\% increase!

However, for a large city of 10 million people

\[
\text{Relative change} = \frac{1000}{10,000,000} = 0.0001
\]

a 0.01\% increase in population.

Relative change

Suppose a quantity changes from \( P_0 \) to \( P_1 \), then

\[
The \ relative \ change \ in \ a \ quantity \ P = \frac{\text{Change in } P}{P_0} = \frac{P_1 - P_0}{P_0}.
\]

**Note:**

1. The relative change has no units
2. Maybe expressed as a percentage.
Suppose at time $t$, a particle's position is given by $s(t)$.

<table>
<thead>
<tr>
<th>$t$ (s)</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>10</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s(t)$ (m)</td>
<td>0</td>
<td>72</td>
<td>92</td>
<td>144</td>
<td>188</td>
</tr>
</tbody>
</table>

Find the average velocity of the particle from $t = 8$ to $t = 10$.

Average velocity is the average rate of change of distance w.r.t. time.

$$
\text{Average velocity} = \frac{\text{Change in distance}}{\text{Change in time}} = \frac{144 - 72}{10 - 3} = \frac{72}{7} = 10.28 \text{ m/s}.
$$

$[3, 10]$  

$M_1 > M_2$ so the average velocity is greater on $[0, 1]$ compared to $[1, 2]$. 

$M_1$ - average velocity on $[0,1]$  

$M_2$ - average velocity on $[1,2]$
When the price is $1.00, a store sells 3,000 items, when the price goes up to $1.25, the quantity sold drops to 2,700.

Relative change in price of items = \[ \frac{1.25 - 1.00}{1.00} = 0.25 \]. (25%)

Relative change in quantity sold = \[ \frac{2,700 - 3,000}{3,000} = -0.1 \]. (10%)

We can compute the ratio (in absolute terms)

\[
\left| \frac{\text{Relative change in quantity}}{\text{Relative change in price}} \right| = \left| \frac{-0.1}{0.25} \right| = \frac{10}{25} = 0.4 .
\]

so the number of items sold decreases by 0.4% when the price drops.

Increases by 1%.

This ratio is called the elasticity (measures how much changes in price affects quantity sold).

They do

\[ # 25 \]

\[ # 55 \]