1.4 Applications to Economics

Cost Function

$C(q)$ – Total cost of producing a quantity, q of goods.

Type of costs

1. FIXED COSTS – costs incurred even when $q=0$ (eg rental costs, labor)
2. VARIABLE COSTS – depend on the number of items produced.

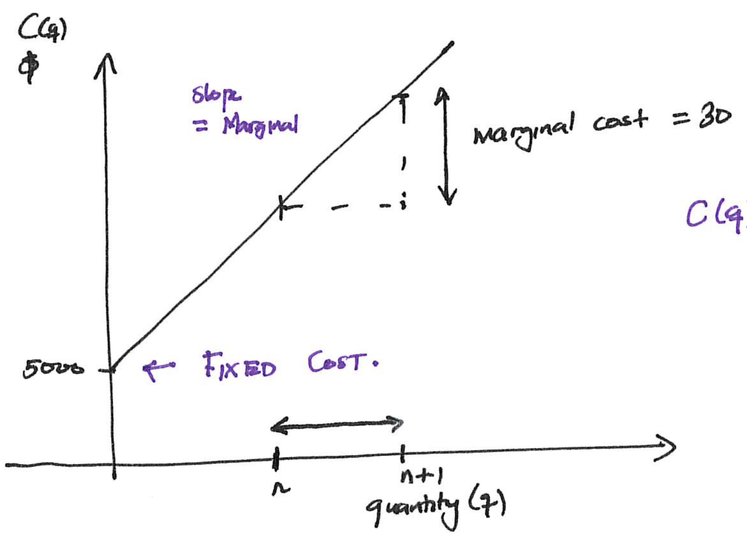
Example #1

A company makes chairs at a fixed cost of \$5000 with variable costs of \$30 per chair.

$$\boxed{C(q) = \text{FIXED COSTS} + \text{Variable Costs}}$$

$$C(q) = 5000 + 30q$$

MARGINAL COST – This is the variable cost for one additional unit.



$C(q) = 5000 + 30q$

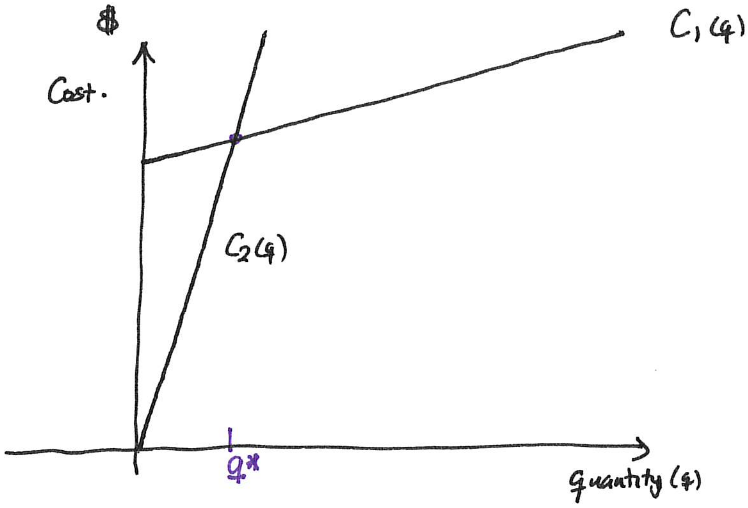
Marginal cost are \$30/chair

In general, if $C(q)$ is a linear function,

1. The Marginal cost is the slope of the cost function.
2. FIXED costs are the vertical intercept.

Examples of cost functions

(a) Fixed Costs are large but marginal costs are small C_1
(b) No fixed costs but high marginal costs. C_2



Which one is better?

It depends on how much you are willing to produce!
for $q < q^*$, C_2 is better but for $q > q^*$ C_1 is better.

Revenue Function

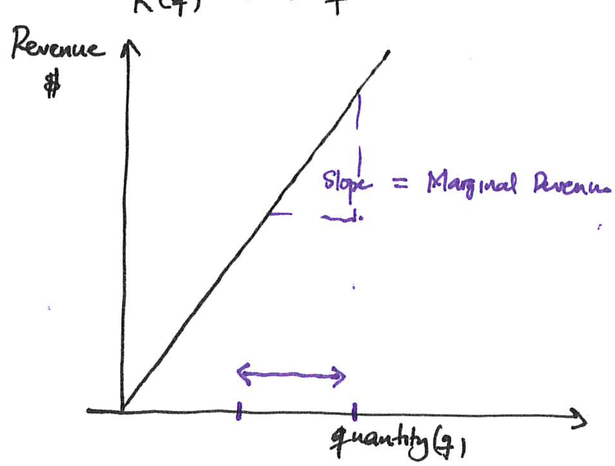
- Total revenue recieved by a firm from selling, q, of some product

$$\text{Revenue} = \text{Price} \times \text{Quantity}$$

$$R(q) = pq.$$

In our chair Example, if the company sells each chair for \$50, then

$$R(q) = 50q$$

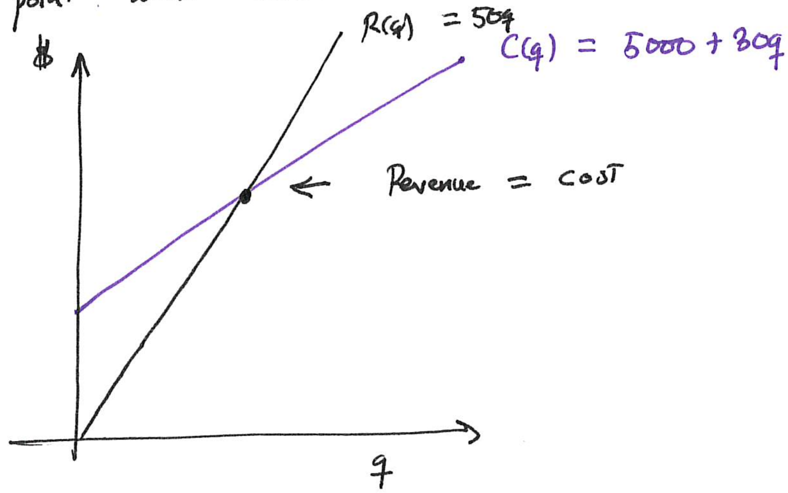


Marginal revenue

Revenue obtained from selling one additional product $= \$50/\text{item}.$

BREAK-EVEN POINT

This is the point where revenue = cost



Calculating the break-even point

$$\text{REVENUE} = \text{COST}$$

$$50q = 5000 + 30q$$

$$20q = 5000 \Rightarrow q = 250 \text{ chairs.}$$

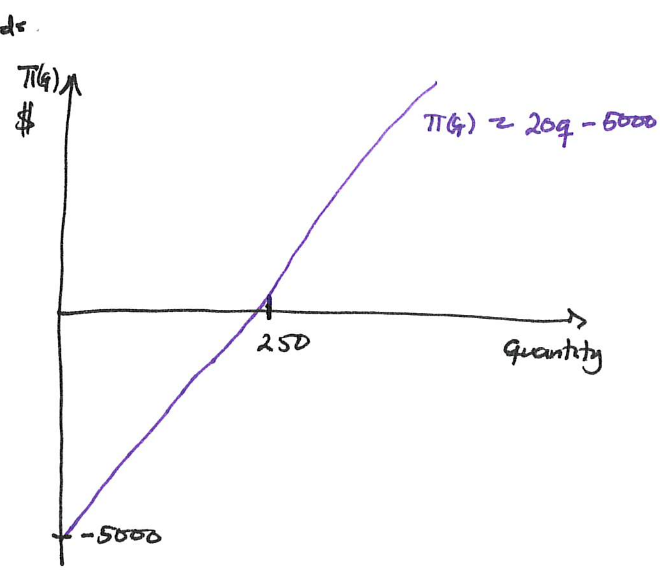
PROFIT ($\pi(q)$)

$$\text{PROFIT} = \text{REVENUE} - \text{COST}$$

$\pi(q)$ – profit earned from selling q goods.

$$\pi(q) = 50q - (5000 + 30q)$$

$$= \cancel{500}\ 20q - 5000$$



Marginal Profit

– Profit earned from selling one additional item $= \$20/\text{item}.$

Depreciation Function –

A new bus is worth \$100,000 in 2010 depreciates linearly to 20,000 in 2030.

We can define a depreciation function that describes the value of the asset over time (since 2010).

$$V - V_0 = m(t - t_0).$$

$$m = \frac{\text{change in value}}{\text{change in time}} = \frac{\$20,000 - \$100,000}{20 \text{ year}} = -\frac{\$80,000}{20\text{yrs}}$$

$$= -4,000 \text{ per year.}$$

$$V - 100,000 = -4.000\,(t - 0)$$

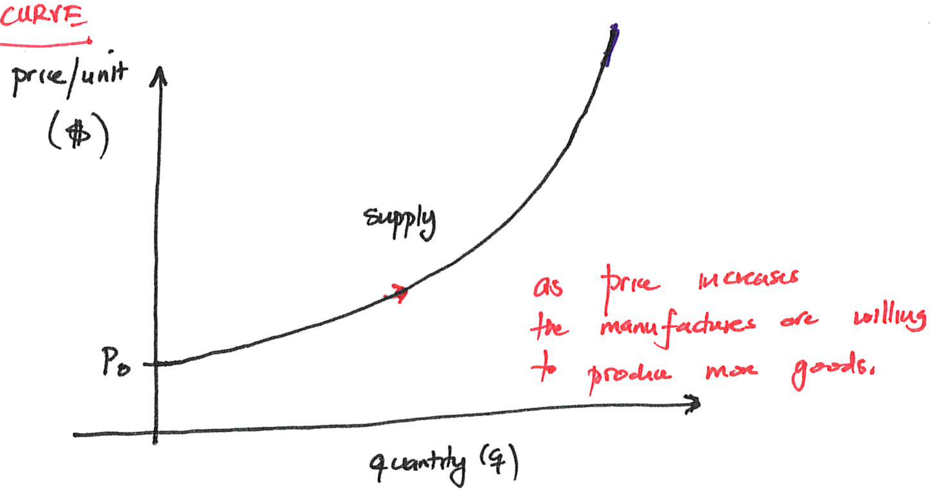
$$\boxed{V(t) = 100,000 - 4.000t.}$$

(Use slope intercept form!)

Supply and Demand Curves

The quantity, q of a product produced depends on the price, p. i.e as price increases manufacturers are willing to supply more, however the quantity demanded by consumers decreases.

Supply Curve

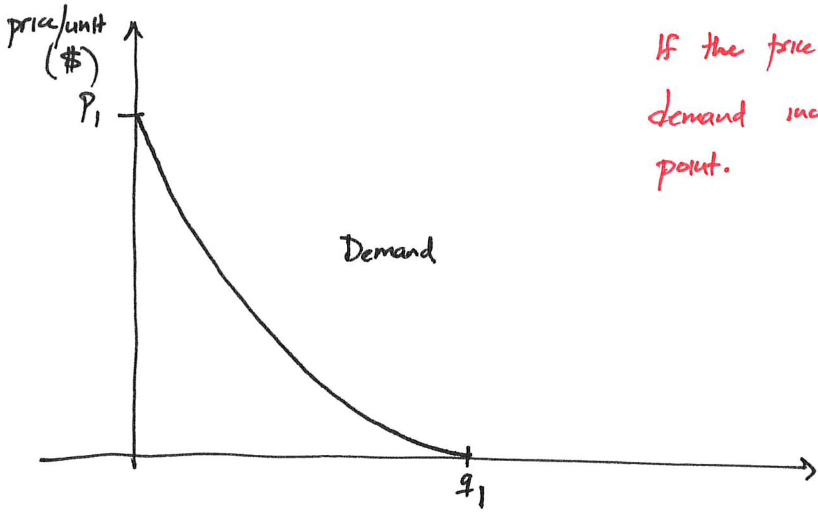


The supply curve relates the quantity, q of an item that manufacturers are willing to make per unit time to the price, p.

- For prices less than or equal to P_0, manufactures are not willing to produce.

DEMAND CURVE

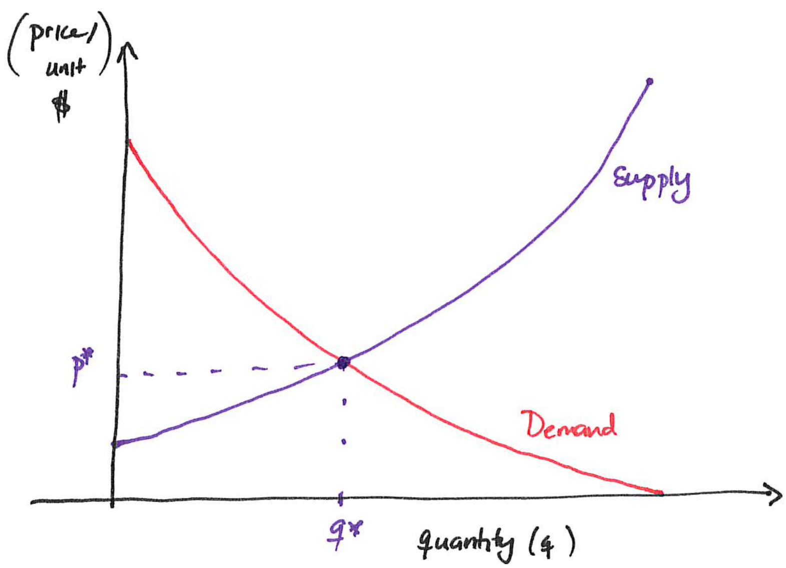
The quantity, q, of an item demanded by consumers per unit time to the price p



If the price decreases, consumer demand increases upto a saturation point.

- At prices greater than or equal to P_1, consumers are not willing to buy any goods.
- q_1 is the quantity demanded when the product is free.

Equilibrium Price and Quantity



In theory the forces of supply and demand result in the market settling on an equilibrium price (p^*, q^*).

Example

Suppose the quantity supplied $q = 3p - 50$ and quantity demanded $= 100 - 2p$.

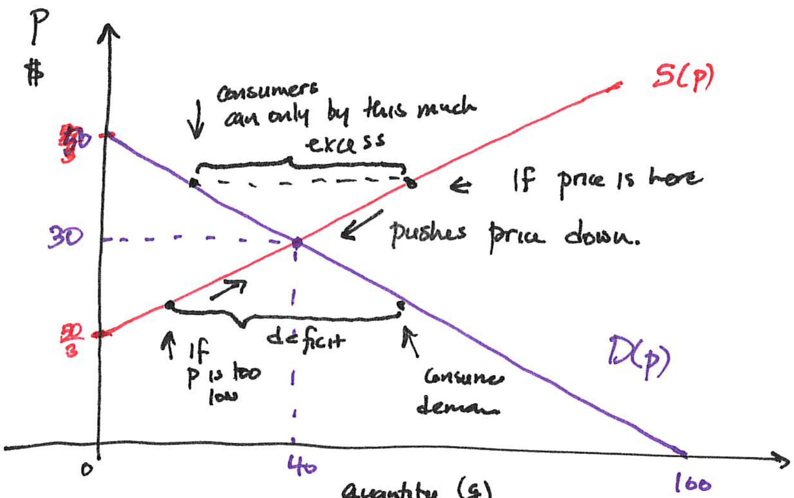
We can compute the equilibrium price and quantity

SUPPLY = DEMAND

$$3p - 50 = 100 - 2p$$

$$5p = 150 \Rightarrow p = \frac{150}{5} = \$30$$

Equilibrium quantity $q = 3p - 50 = 3 \cdot 30 - 50 = 40$ items.



for historical reasons price is on the price is on the vertical axis.

Demand :

$q = 100 - 2p$

$p = 50$ @ $q = 0$

Supply:

$q = 3p - 50$

@ $q = 0 \Rightarrow 0 = 3p - 50$, $p = \frac{50}{3}$

$50 = 3p$

@ $p = 0$

DISCUSSION

- If price is above equilibrium, there is an excess of products in the market thus pushing prices down (suppliers are wasting money)
- If the price is too low, there is a shortage, pushing prices up. (suppliers make more money by increasing the price and quantity)

THE EFFECT OF TAXES ON EQUILIBRIUM

Example.

In the example above, suppose a tax of \$5.00 is imposed on the suppliers. How does this affect the equilibrium price and quantity?

1. The consumers pay p, but the supplier only recieves $p - 5$ dollars so the new Quantity supplied is

$$3 \cdot \text{amount recieved} - 50 = 3(p-5) - 50$$
$$= 3p - 15 - 50$$
$$= 3p - 65 \quad (\text{NEW Supply Equation})$$

2. The quantity demanded remains unchanged because the price is still p.

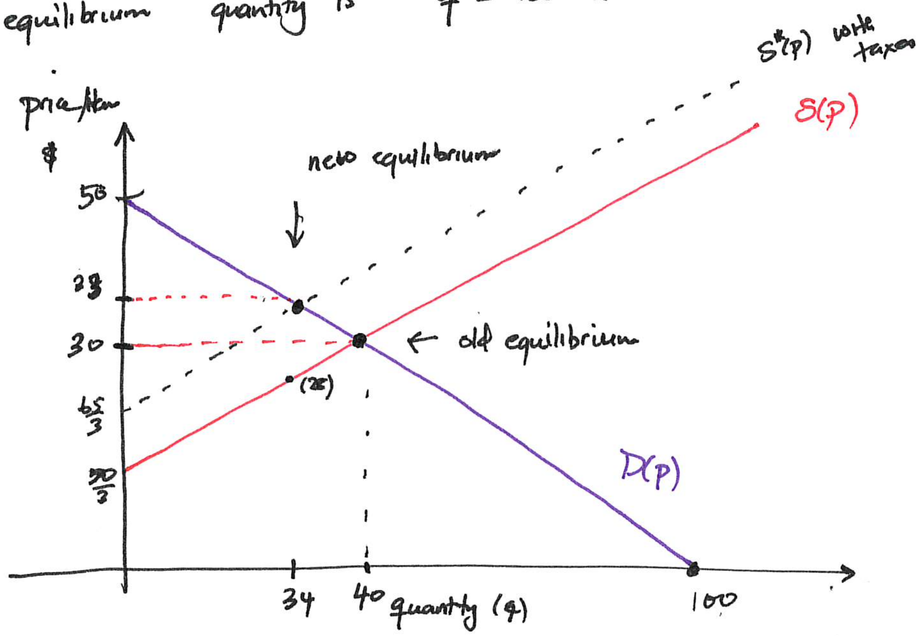
$$D(p) = 100 - 2p.$$

3. We find the new equilibrium price and quantity by setting the demand = supply

$$100 - 2p = 3p - 65$$

$$165 = 5p \quad \Rightarrow \quad p = \frac{165}{5} = \$33$$

The equilibrium quantity is $q = 100 - 2 \cdot 33 = 34$ items.



$S(p) = 3p - 50$

$S^*(p) = 3p - 65$

new vertical int

$0 = 3p - 65$

$p = \frac{65}{3}$

$21\frac{2}{3}$

NOTE

Even though the ~~price~~ tax ~~increase~~ was $5.00, the consumer only pays $3.00 more.

The supplier pays the additional $2.00 and only retains $28.00.

BUDGET CONSTRAINTS

A taxi company has an annual budget of $720,000 to spend on drivers and car replacements.

Drivers cost $30,000 each and each replacement costs $20,000.

Suppose d is the number of drivers and c is the number of car replacements.

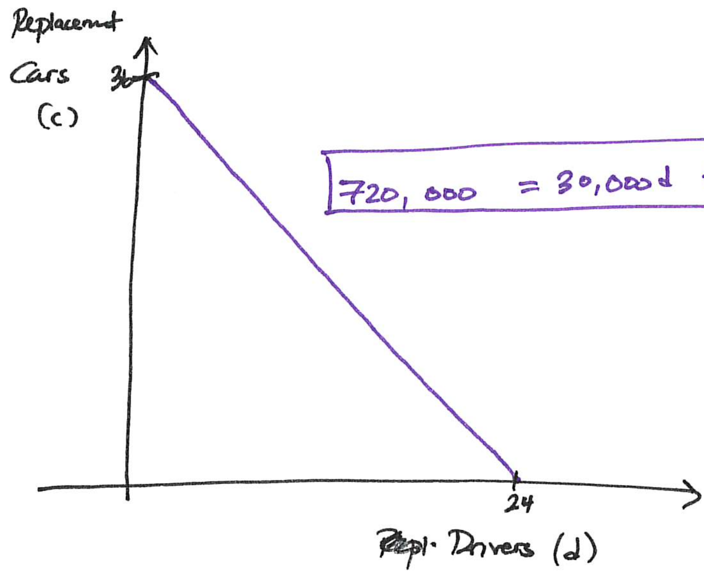
Total cost of drivers = $30{,}000d$

Total cost of replacements = $20{,}000c$

BUDGET = Total cost of drivers + Total cost of replacements

$$\boxed{720{,}000 = 30{,}000d + 20{,}000c}$$

(This is called the Budget constraint)



$c = 0$ (i.e. no replacement)

$720{,}000 = 30{,}000d$

$\frac{720{,}000}{30{,}000} = d \Rightarrow d = \frac{72}{3}$

$= 24$ drivers

$d = 0$ (no drivers)

$720{,}000 = 20{,}000c$

$\frac{720{,}000}{20{,}000} = c$

$c = \frac{72}{2} = \underline{36}$

To stay on budget, the company should stay on the line.

<u>They do</u>

1. Cost, Revenue, Profit function
2. Equilibrium Supply & Quantity

A company with fixed costs of \$6000 and marginal costs of \$10/item sells goods at a price of 12 each

(a) Write down the revenue and cost function
(b) Find the break even point & illustrate it graphically