Cost Function

\[ C(q) \] - Total cost of producing a quantity, \( q \) of goods.

Type of costs

1. **Fixed Costs** - costs incurred even when \( q = 0 \) (e.g., rental costs, labor)
2. **Variable Costs** - depend on the number of items produced.

Example #1

A company makes chairs at a cost of $5000 with variable costs of $80 per chair.

\[ C(q) = \text{Fixed Costs} + \text{Variable Costs} \]

\[ C(q) = 5000 + 80q \]

**Marginal Cost** - This is the cost for one additional unit.

Marginal costs are $80/chair.

In general, if \( C(q) \) is a linear function,

1. The Marginal cost is the slope of the cost function.
2. Fixed costs are the vertical intercept.
Examples of cost functions:

(a) Fixed costs are large but marginal costs are small \( C_1 \)

(b) No fixed costs but high marginal costs \( C_2 \)

\[ \text{Cost} \]
\[ \text{Quantity (q)} \]

What one is better?

It depends on how much you are willing to produce!

for \( q < q^* \), \( C_2 \) is better but for \( q > q^* \), \( C_1 \) is better.

**Revenue Function**

Total revenue received by a firm from selling \( q \) of some product

\[ \text{Revenue} = \text{Price} \times \text{Quantity} \]

\[ R(q) = pq \]

In our chair example, if the company sells each chair for \( \$50 \), then

\[ R(q) = 50q \]
Marginal revenue

Revenue obtained from selling one additional product = $50/1 km.

Break-even point

This is the point where revenue = cost

\[ R(q) = 50q \]
\[ C(q) = 5000 + 80q \]

\[ \text{Revenue} = \text{cost} \]

Calculating the break-even point

\[ R(q) = C(q) \]
\[ 50q = 5000 + 80q \]
\[ 20q = 5000 \implies q = 250 \text{ chairs} \]

Profit \((\Pi(q))\)

Profit = Revenue - Cost

\[ \Pi(q) = \text{profit earned from selling } q \text{ goods} \]

\[ \Pi(q) = 50q - (5000 + 80q) \]
\[ = 20q - 5000 \]

Marginal Profit

- Profit earned from selling one additional item = $20/1 km.
A new bus is worth $100,000 in 2010 and depreciates linearly to 20,000 in 2030.

We can define a depreciation function that describes the value of the asset over time (since 2010).

\[ V - V_0 = m(t - t_0). \]

\[ m = \frac{\text{change in value}}{\text{change in time}} = \frac{\$20,000 - \$100,000}{20 \text{ years}} = \frac{-\$80,000}{20 \text{ years}} = -4,000 \text{ per year}. \]

\[ V - 100,000 = -4,000(t - 0) \]

\[ V(t) = 100,000 - 4,000t. \] (Use slope intercept form!)

**Supply and Demand Curves**

The quantity, \( q \), of a product produced depends on the price, \( p \). i.e., as price increases, manufacturers are willing to supply more, however, the quantity demanded by consumers decreases.

**Supply Curve**

The supply curve relates the quantity, \( q \), of an item that manufacturers are willing to make per unit time to the price, \( p \). For prices less than or equal to \( p_0 \), manufacturers are not willing to produce.
Demand Curve

The quantity, q, of an item demanded by consumers per unit time to the price p.

If the price decreases, consumer demand increases up to a saturation point.

- At prices greater than or equal to \( p_1 \), consumers are not willing to buy any goods.
- \( q_1 \) is the quantity demanded when the product is free.

Equilibrium Price and Quantity

In theory, the forces of supply and demand result in the market settling on an equilibrium price \((p^*, q^*)\).
Example

Suppose the quantity supplied \( q = 3p - 50 \) and quantity demanded \( = 100 - 2p \).

We can compute the equilibrium price and quantity

Supply = Demand

\[ 3p - 50 = 100 - 2p \]

\[ 5p = 150 \Rightarrow p = \frac{150}{5} = 30 \]

Equilibrium quantity \( q = 3p - 50 = 3 \times 30 - 50 = 40 \) items.

\[ \text{Diagram:} \quad \text{Demand:} \quad q = 100 - 2p \]

\[ p = 50 \quad \text{at} \quad q = 0 \]

Discussion

- If price is above equilibrium, there is an excess of products in the market, thus pushing prices down (suppliers are pushing money).
- If the price is too low, there is a shortage, pushing prices up.
  (suppliers make more money by increasing the price and quantity)

**The EFFECT OF TAXES ON EQUILIBRIUM**

Example.

In the example above, suppose a tax of \( $5.00 \) is imposed on the suppliers.

How does this affect the equilibrium price and quantity?

1. The consumers pay \( p \), but the supplier only receives \( p - 5 \) dollars, so

the new quantity supplied is

\[ 3 \times \text{amount required} - 50 = 3(p - 5) - 50 \]

\[ = 3p - 15 - 50 \]

\[ = 3p - 65 \quad \text{(NEW Supply Equation)} \]
2. The quantity demanded remains unchanged because the price is still $p$.

\[ D(p) = 100-2p. \]

3. We find the new equilibrium price and quantity by setting the demand = supply.

\[ 100 - 2p = 3p - 65 \]
\[ 165 = 5p \quad \Rightarrow \quad p = \frac{165}{5} = 33 \]

The equilibrium quantity is $q = 100 - 33 = 34$ items.

- \[ S(p) \text{ with tax} \]
- \[ S'(p) = 3p - 50 \]
- \[ S''(p) = 3p - 65 \]
- New vertical int $b = 3p - 65$
- $p = \frac{65}{3} \approx 21.67$

**Note:**
Even though the tax raises $5.00, the consumer only pays $3.00 more.

The supplier pays the additional $2.00 and only retains $28.00.

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**Budget Constraints**

A taxi company has an annual budget of $720,000 to spend on drivers and car replacements.

Drivers cost $30,000 each and each replacement costs $20,000.

Suppose $d$ is the number of drivers and $c$ is the number of car replacements.

Total cost of drivers = $30,000d$
Total cost of replacements = $20,000c$
BUDGET = Total cost of drivers + Total cost of replacements

720,000 = 30,000d + 20,000c  
(This is called the Budget constraint)

Replace
Cars (c)

720,000 = 20,000d + 20,000c

Replace Drivers (d)

c=0 (i.e. no replacement)

720,000 = 30,000d

\[
\frac{720,000}{30,000} = d \Rightarrow d = \frac{72}{3} = 24 \text{ drivers}
\]

d=0 (no drivers)

\[
\frac{720,000}{20,000} = c \\
\frac{72}{2} = 36
\]

To stay on budget, the company should stay on the line.

They do
1. Cost, Revenue, Profit function
2. Equilibrium, Supply & Quantity

A company incurs fixed costs of $6000 and marginal costs of $10/km sells goods at a price of 8 1/2 each
(a) Write down the revenue and cost functions
(b) Find the break even point & illustrate it graphically