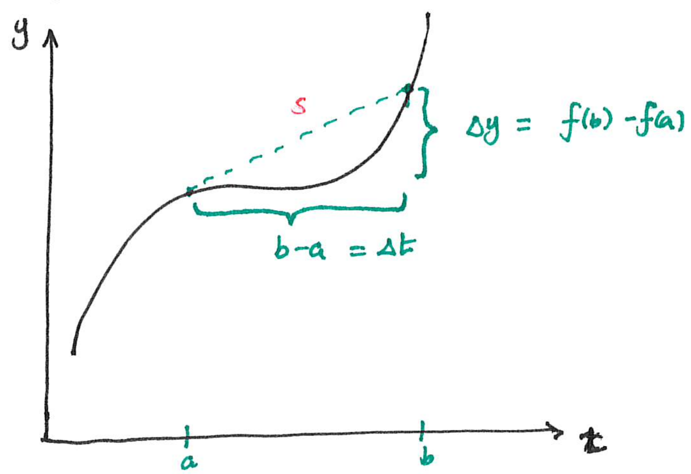
Chapter 2    2.1    

## Instantaneous rate of change

Given $y = f(t)$, the average rate of change between $t = a$ and $t = b$ is

$$\frac{f(b) - f(a)}{b - a} = \frac{\Delta y}{\Delta t}$$

Graphically,




The average rate of change is the slope of s.

## Average Velocity

(Above, if $f(t)$ is a distance function, s is the average velocity)

- Average rate of change of distance

e.g If you travel 100miles in 2hrs, average velocity = $\dfrac{100 - 0 \text{ miles}}{(2-0) \text{ hrs}}$ = 50miles/hr.

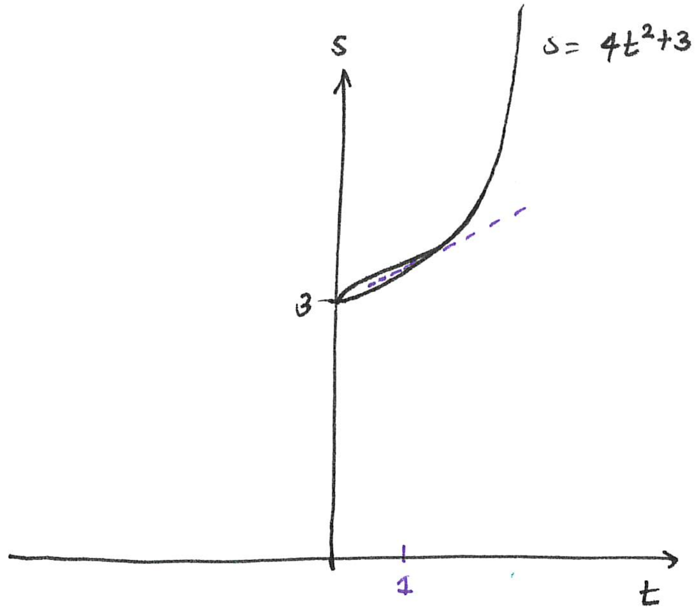
However the velocity at any given time will vary.

Calculate the average velocity over smaller time intervals

## Instantaneous velocity

Suppose a particle moves a distance of $s = 4t^2 + 3$.

Estimate the instantaneous velocity at $t = 1$




Estimate @ $\underline{t=1}$
Instantaneous velocity

Find the average velocity for smaller and smaller time intervals starting @ $t=1$.

[1, 1.01]

$$\frac{f(1.01) - f(1)}{1.01 - 1.00} = \frac{4(1.01)^2+3 - 7}{1.01} = 8.04 \text{ m/s}$$

[1, 1.001]

$$\frac{f(1.001) - f(1)}{1.001 - 1} = 8.004 \text{ m/s}$$

[1, 1.0001]

$$\frac{f(1.0001) - f(1)}{1.0001 - 1} = 8.00039 \text{ m/s}$$

Instantaneous velocity $\rightarrow 8$.

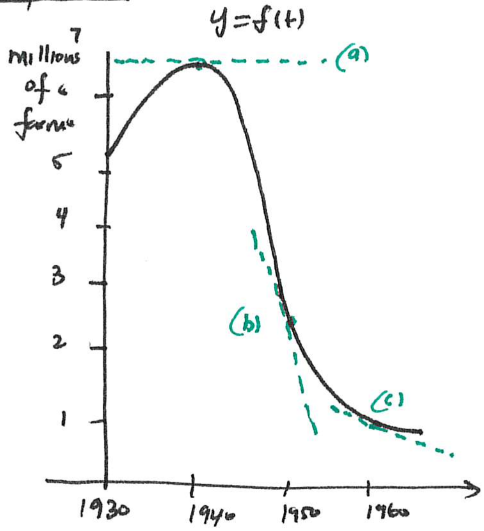
Instantaneous velocity

is the limit of the average velocity of an object over shorter and shorter time intervals.

- Instantaneous rate of change of $y = f(t)$ is the ~~instantaneous rate~~ limit of the average rates of change of $f$ over shorter time intervals.

This is the slope of the tangent line to the function at $t=a$.

## Example #1




(a) In 1940, the rate of change of the number of farms is ~~constant~~ zero i.e the number of farms is constant

(b) In 1950, the number of farms is decreasing

(c) In 1940, the rate of change is decreasing but not as fast as in 1950.

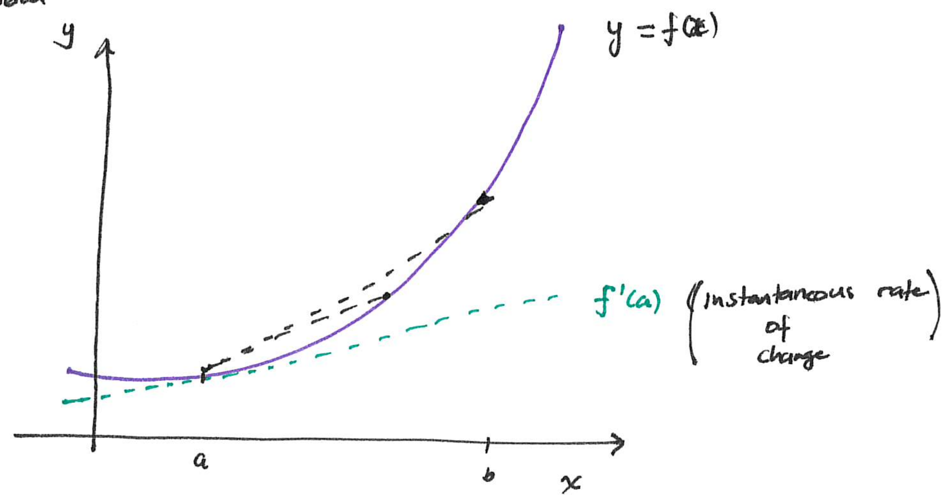
## Derivative at a point.

The instantaneous rate of change of $f$ at a point $x=a$ is the derivative of $f$ at $x=a$.

Notation $\quad f'(a) \quad$ ( units : $\rightarrow \dfrac{\text{units of } y}{\text{units of } x}$

In our previous example, $f(t) = 4t^2+3$, we say $f'(1)=8$.

In general




$f'(a)$ = slope of the tangent line to the curve at $x=a$.