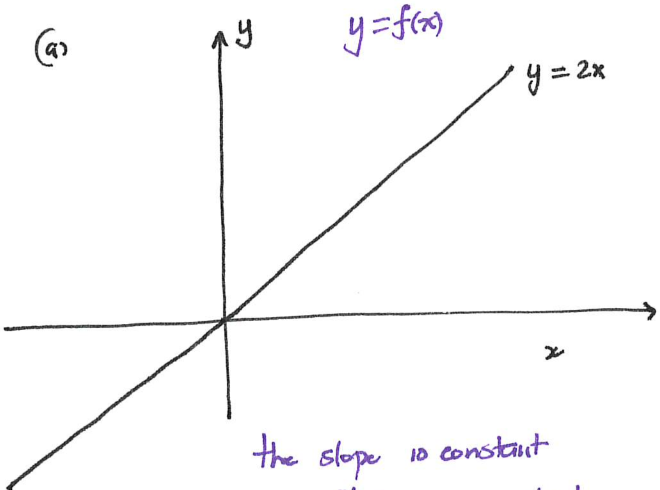
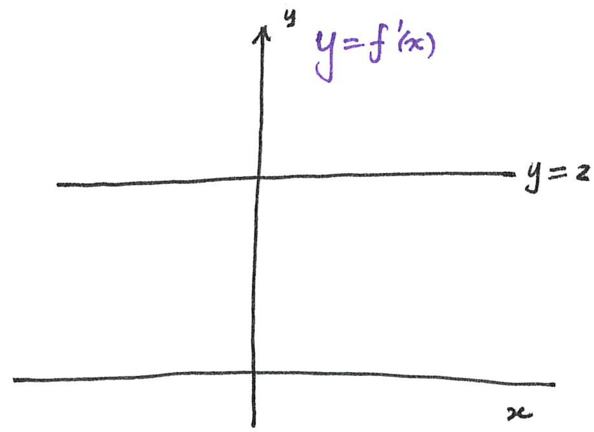
THE DERIVATIVE FUNCTION 2.2

Given $y = f(x)$ we can define $y = f'(x)$ – the derivative function of $y = f(x)$.

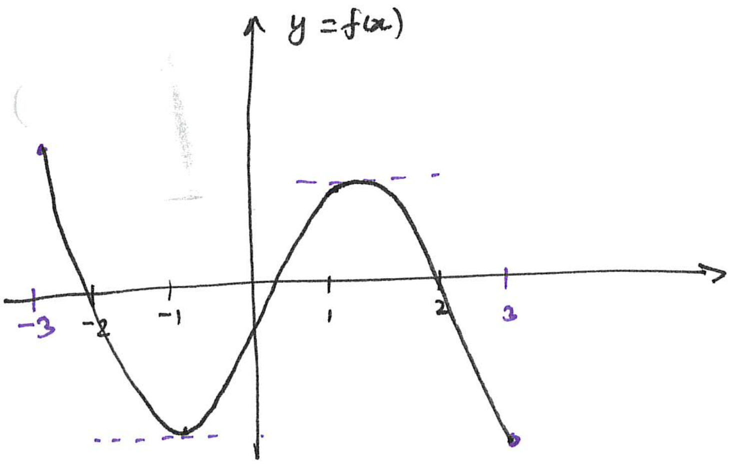
Example #1

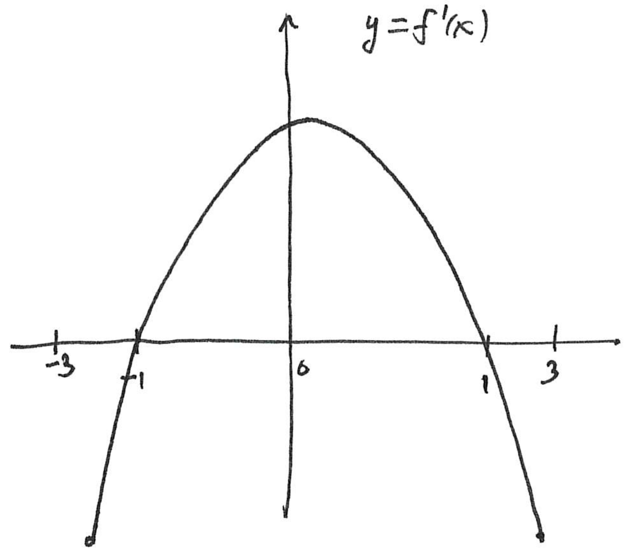
(a)





the slope is constant so $f'(x)$ is a constant.





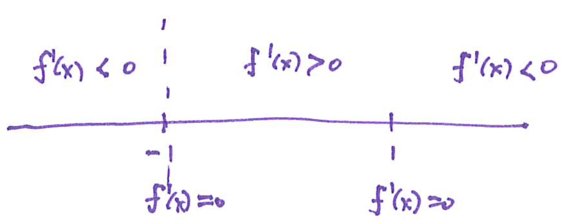
Note that

1. The tangent line to $y = f(x)$ is horizontal @ $x = -1, 1$
   so $f'(-1) = f'(1) = 0$

2. $[-3, -1)$ – $f'(x) < 0$ i.e $f(x)$ is decreasing

3. $[-1, 1]$ – $f'(x) > 0$ i.e $f(x)$ is ~~decreasing~~ increasing

4. $(1, 3]$ – $f'(x) < 0$ i.e $f(x)$ is decreasing



Graphical interpretation of $y = f'(x)$

1. If $f'(x) > 0$ on an interval, then f is <u>increasing</u> on that interval
2. If $f'(x) < 0$ on an interval, then f is <u>decreasing</u> on that interval
3. If $f'(x) = 0$ on an interval, then f is <u>constant</u> on that interval.

Estimating $f'(x)$ from data

The table below gives values of concentration (mg/cc) of drug in the blood stream at time t (mins)

t(mins)	0	0.1	0.2	0.3	0.4
C(t) mg/cc	0.84	0.89	0.94	0.98	1.0

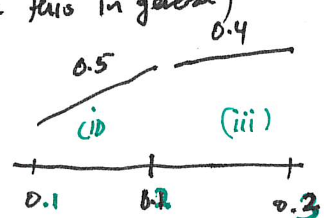
We can estimate $c'(t)$ – the rate of change of the concentration w.r.t time.

e.g (i) $c'(0) \cong \dfrac{c(0.1) - c(0)}{0.1 - 0} = \dfrac{0.89 - 0.84}{0.1} = \dfrac{0.05}{0.1} = 0.5$ mg/cc per min

(ii) $c'(0.1) \cong \dfrac{c(0.2) - c(0.1)}{0.2 - 0.1} = \dfrac{0.94 - 0.89}{0.1} = 0.5$ mg/cc per min

(iii) $c'(0.2) \cong \dfrac{c(0.3) - c(0.2)}{0.3 - 0.2} = \dfrac{0.98 - 0.94}{0.1} = 0.4$ mg/cc per min.

<u>Improving estimates</u> (Use this in general)

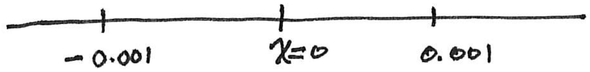


We can average the slopes

$$c'(0.2) \cong \frac{1}{2}\left(\text{slope on left} + \text{slope on right}\right) = \frac{0.5 + 0.4}{2} = 0.45.$$

Given $R(x) = 160(1.1)^x$

Estimate $R'(0)$ by computing the average rate of change near $x = 0$. using $\Delta x = 0.001$.



Average rate of change Left $= \dfrac{R(0) - R(-0.001)}{0 - (-0.001)}$

Average rate of change right $= \dfrac{R(0.001) - R(0)}{0.001 - 0}$

Average rate of change $= \dfrac{1}{2}\left(\text{left average rate of change} + \text{right average rate of change}\right) = 9.531$