Chapter 3 (Derivative formula (3.1))

Derivative of a function ~ slope / Rate of change

(i) Constant functions

If $f(x) = k$, then $f'(x) = 0$.

(ii) Linear functions

If $f(x) = mx+b$, then $f'(x) = m$

(iii) POWER functions

If $f(x) = x^n$, for any real number n, then

$$f'(x) = nx^{n-1}$$

e.g

$f(x) = 5x+3$, $f'(x) = 5$ or $\frac{d}{dx}\left[5x+3\right] = 5$

$f(x) = x^7$, $f'(x) = 7x^{7-1} = 7x^6$ or $\frac{d}{dx}\left[x^7\right] = 7x^6$.

Combining functions

(i) Constant multiple

$$\frac{d}{dx}\left[c f(x)\right] = c f'(x)$$

(ii) Sum and difference

(a) $\frac{d}{dx}\left[f(x) + g(x)\right] = f'(x) + g'(x)$

(b) $\frac{d}{dx}\left[f(x) - g(x)\right] = f'(x) - g'(x)$.

Example #1

(a) $5\sqrt{x} + \frac{1}{x^3} = f(x)$

Rewrite

$$f(x) = 5x^{\frac{1}{2}} + x^{-3} \Rightarrow f'(x) = 5\cdot\frac{1}{2}x^{\frac{1}{2}-1} + (-3)x^{-3-1}$$

$$= \frac{5}{2}x^{-\frac{1}{2}} - 3x^{-4} = \frac{5}{2\sqrt{x}} - \frac{3}{x^4}$$

(b) Find the slope of the tangent line to the curve $y = x^3 + 2x^2 - 5x + 7$

$$\frac{dy}{dx} = 3x^2 + 4x - 5$$

other notation

@ $x=1$, $m = f'(1) = 3(1)^2 + 4(1) - 5 = 2$ ooo $\left.\frac{dy}{dx}\right|_{x=1} = 2.$

Equation of tangent line, (we have slope and x-coord)

@ $x=1$, $y = 1^3 + 2(1)^2 - 5(1) + 7 = 5$ so $(1,5)$ is a point on the tangent line.

$$y - 5 = 2(x-1) \Rightarrow y = 2x + 3.$$

Example #2.

The cost function, $C(q)$ of producing q items is given by

$$C(q) = 0.08q^3 + 75q + 1000$$

Marginal cost

$$MC(q) = C'(q) = 3\cdot 0.08q^2 + 75$$

$$= 0.24q^2 + 75.$$

$$C'(50) = 0.24(50^2) + 75 = \$675/\text{item}$$

This is the additional cost of producing 1 more chair when 50 chairs are being produced.