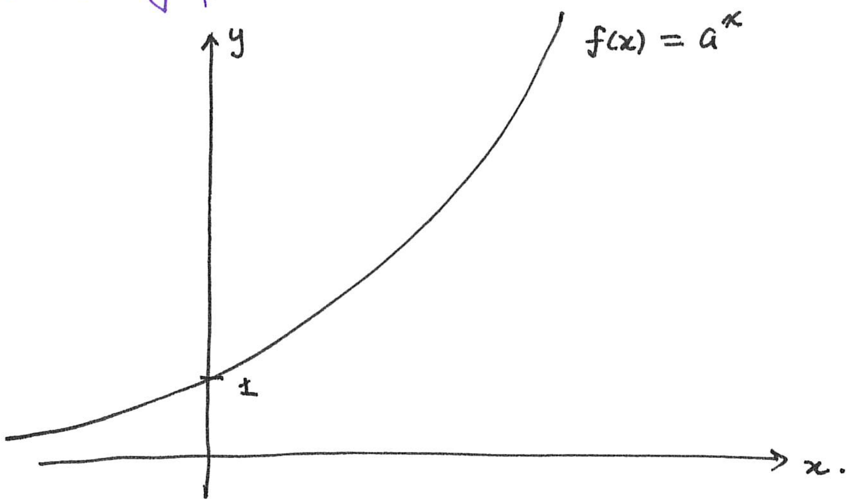
## 3.2

### Exponential and Logarithmic functions.

Recall the graphs




**Rule**

(i) $\frac{d}{dx}(a^x) = \ln(a) \cdot a^x$

If $a = e$, then

$\frac{d}{dx}(e^x) = \ln(e)\, e^x = e^x.$

(ii) $\frac{d}{dx}\left(e^{kx}\right) = \frac{d}{dx}\left[\left(e^k\right)^x\right] = \ln\left(e^k\right) e^{kx} = ke^{kx}$

(iii) $\frac{d}{dx}\left(\ln(x)\right) = \frac{1}{x}.$

**Example**

$y = 4\ln t + 7e^{2t} - 4t^2 + 2^t$

$\frac{dy}{dt} = \frac{4}{t} + 7 \cdot 2e^{2t} - 8t + \ln(2)\, 2^t$

## Example

The demand curve of a product is given by

$$q = f(p) = 10,000\, e^{-0.25p}$$

where $q$ is the quantity sold at a price of $p$ dollars.

Find and interpret

$f(2)$ and $f'(2)$.

(a) $f(2) = 10,000\, e^{-0.25(2)} = 10,000\, e^{-0.5} = 6065$

is the number of items sold when the price is \$2.

(b) $f'(p) = 10,000 \cdot (-0.25 e^{-0.25p})$

$f'(2) = -2500\, e^{-0.25(2)} = -2500\, e^{-0.5} = \underline{-1516}$

When the price is \$2.00, if the price increases by \$1.00, the quantity sold decreases by 1516 items.

## Example #2

Suppose \$1000 is deposited into a bank account that pays 8% annual interest compounded continuously

(a) Find a formula, $f(t)$ for the balance, $t$ years after the initial deposit

(b) Find $f(10)$ and $f'(10)$ and interpret the meaning in terms of money.

(a) $f(t) = 1000\, e^{0.08t}$

(b) $f(10) = 1000\, e^{0.08(10)} =$ \$2225.54 is the balance after 10yrs

(ii) $f'(t) = 1000 \cdot 0.08 e^{0.08t} \Rightarrow f'(10) = 80 e^{0.08(10)} = 178.04$

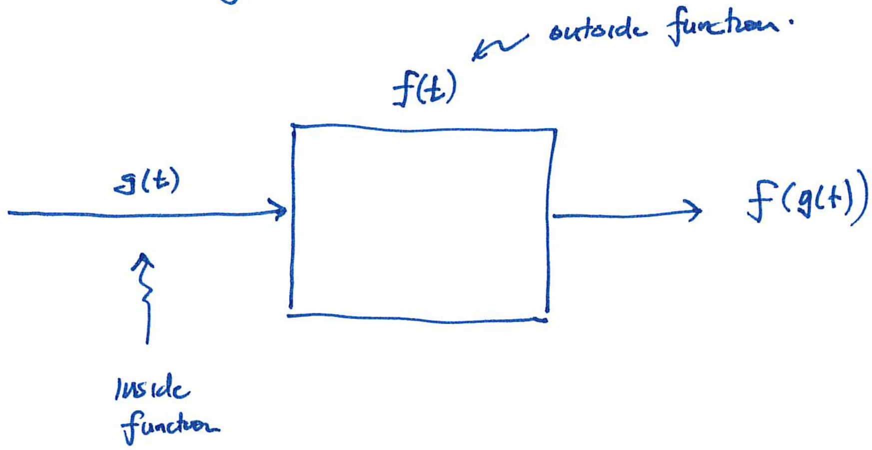
After 10 yrs, The balance is growing at a rate of \$178 per year

## 3.3 Chain Rule : Derivatives of compositions of functions.

### GOAL

Find the derivative of the composition

$$y = f(g(t)).$$




### Examples

(a) $y = (1+t^5)^2 = f(g(t))$

is a composition of $f(t) = t^2$ and $g(t) = (1+t^5)$

(b) $y = \sqrt{t^2+t}$

is a composition of

$f(t) = \sqrt{t}$ and $g(t) = t^2+t$.

(c) $y = e^{x^3}$

is a composition of $f(x) = e^x$

$g(x) = x^3$

### RULE

$$\frac{d}{dt}\left[f(g(t))\right] = \underbrace{f'(g(t))}_{\text{composition of } f' \text{ and } g} \cdot \overbrace{g'(t)}^{\text{derivative of } g}$$

ie Take the derivative of the outside function (leave $g$) and then take the derivative of the inside function.

**Example**

(a) $y = (1+t^5)^2$

$$\frac{dy}{dt} = 2(1+t^5)\cdot\frac{d}{dt}(1+t^5)$$

$$= 2(1+t^5)\cdot 5t^4.$$

(b) $y = \ln(t^2+1)$

$$\frac{dy}{dt} = \frac{1}{t^2+1}\cdot\frac{d}{dt}(t^2+1)$$

$$= \frac{2t}{t^2+1}.$$

(c) $\cancel{f(t)}$ $y = e^{t^3}$

$$\frac{dy}{dt} = e^{t^3}\cdot(3t^2)$$

(d) $f(x) = (\ln(x))^3$

$f(x) = x^3$ $\quad g(x) = \ln(x)$

$f'(x) = 3x^2$ $\quad g'(x) = \frac{1}{x}$

$$\frac{dy}{dx} = f'(g(x))\cdot g'(x)$$

$$3(\ln(x))^2\cdot\frac{1}{x}.$$