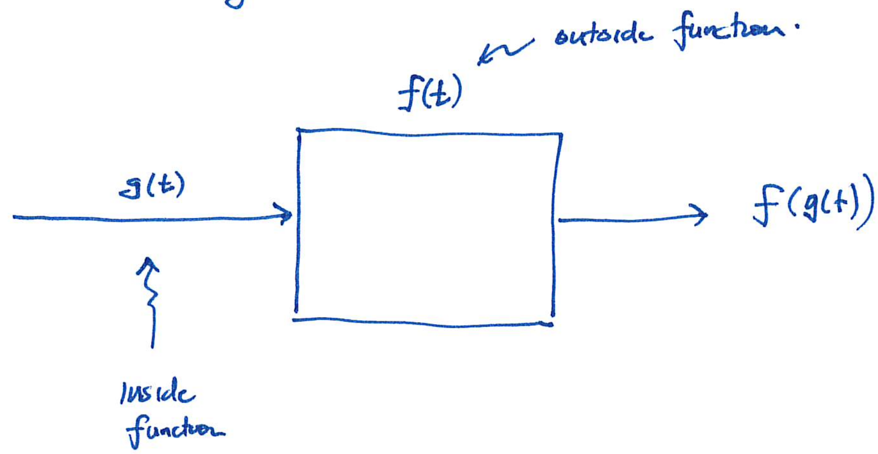
After 10 yrs, The balance is growing at a rate of \$178 per year

3.3 Chain Rule : Derivatives of compositions of functions.

Goal

Find the derivative of the composition

$$y = f(g(t)).$$



Examples

(a) $y = (1+t^5)^2 = f(g(t))$

is a composition of $f(t) = t^2$ and $g(t) = (1+t^5)$

(b) $y = \sqrt{t^2+t}$

is a composition of

$f(t) = \sqrt{t}$ and $g(t) = t^2+t$.

(c) $y = e^{x^3}$

is a composition of $f(x) = e^x$

$g(x) = x^3$

Rule

$$\frac{d}{dt}\left[f(g(t))\right] = \underbrace{f'(g(t))}_{\text{composition of } f' \text{ and } g} \cdot \overbrace{g'(t)}^{\text{derivative of } g}$$

ie Take the derivative of the outside function (leave g) and then take the derivative of the inside function.

Example

(a) $y = (1+t^5)^2$

$$\frac{dy}{dt} = 2(1+t^5) \cdot \frac{d}{dt}(1+t^5)$$

$$= 2(1+t^5) \cdot 5t^4.$$

(b) $y = \ln(t^2+1)$

$$\frac{dy}{dt} = \frac{1}{t^2+1} \cdot \frac{d}{dt}(t^2+1)$$

$$= \frac{2t}{t^2+1}.$$

(c) ~~f(t)~~ $y = e^{t^3}$

$$\frac{dy}{dt} = e^{t^3} \cdot (3t^2)$$

(d) $y(x) = (\ln(x))^3$

$f(x) = x^3$ $\quad g(x) = \ln(x)$

$f'(x) = 3x^2$ $\quad g'(x) = \frac{1}{x}$

$$\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$$

$$3(\ln(x))^2 \cdot \frac{1}{x}.$$