3.4 Product Rule

$$(fg)' = f'g + g'f$$

Example #1

(a) $x^3 e^{3x}$

$f(x) = x^3$, $g(x) = e^{3x}$

$f'(x) = 3x^2$, $g'(x) = 3e^{3x}$

$$\frac{d}{dx}\left(x^3 e^{3x}\right) = 3x^2 \cdot e^{3x} + 3e^{3x} \cdot x^3$$

(b) $t^3 \ln(t^2+1)$

$f(t) = t^3$ $\quad g(t) = \ln(t^2+1)$

$f'(t) = 3t^2$ $\quad g'(t) = \frac{1}{t^2+1} \cdot 2t$ (Chain Rule)

$$\frac{d}{dt}\left(t^3 \ln(t^2+1)\right) = t^3 \cdot \frac{2t}{t^2+1} \cdot t \; 3t^2 \ln(t^2+1) + \frac{2t}{t^2+1} \cdot t^3$$

Quotients

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

Quotient Rule.

$$\frac{d}{dx}\left(\frac{3x+x^2}{5+x}\right)$$

Quotient

$f(x) = 3x + x^2$ $\quad$ $g(x) = 5+x$

$f'(x) = 3+2x$ $\quad$ $g'(x) = 1$

$$\frac{d}{dx}\left(\frac{3x+x^2}{5+x}\right) = \frac{(5+x)(3+2x) - (3x+x^2)\cdot 1}{(5+x)^2}$$

$$\frac{d}{dx}\left(\frac{x}{1+\ln(x)}\right)$$

Quotient Rule

$f(x) = x$ $\quad$ $g(x) = 1+\ln(x)$

$f'(x) = 1$ $\quad$ $g'(x) = \frac{1}{x}$

$$\frac{d}{dx}\left(\frac{x}{1+\ln(x)}\right) = \frac{1+\ln(x)\cdot 1 - x\cdot\frac{1}{x}}{(1+\ln(x))^2}$$

$$= \frac{1+\ln(x)-1}{(1+\ln(x))^2} = \frac{\ln(x)}{(1+\ln(x))^2}$$

(c) $\dfrac{e^x}{x^3}$

$f(x) = e^x$ $\qquad$ $g(x) = x^3$

$f'(x) = e^x$ $\qquad$ $g'(x) = 3x^2$

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

$$= \frac{x^3 \cdot e^x - e^x \cdot 3x^2}{(x^3)^2} = \frac{x^3 e^x - e^x \cdot 3x^2}{x^6}$$