Chapter 4

Application of the Derivative

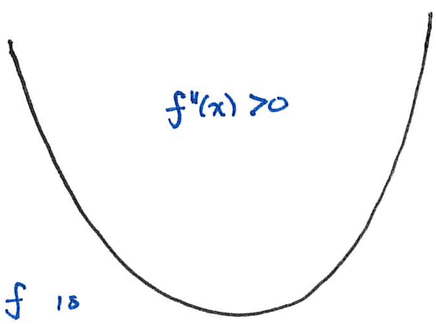
Local maxima and minima

Recap

1. If $f'(x) > 0$ on an interval, then f is increasing on that interval
2. If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

Second derivative

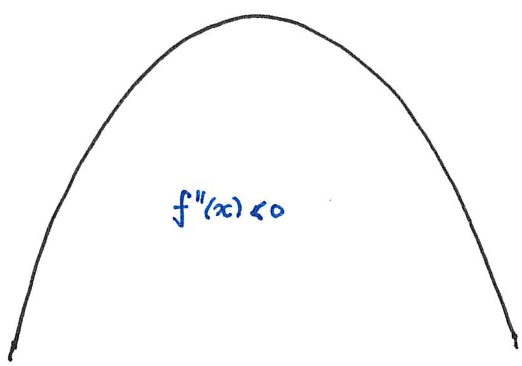
Concave up



f' is increasing $\Rightarrow f''(x) > 0$

If $f''(x) > 0$, f is concave up.

Concave down



f' is decreasing $\Rightarrow f''(x) < 0$

If $f''(x) < 0$, f is concave down.

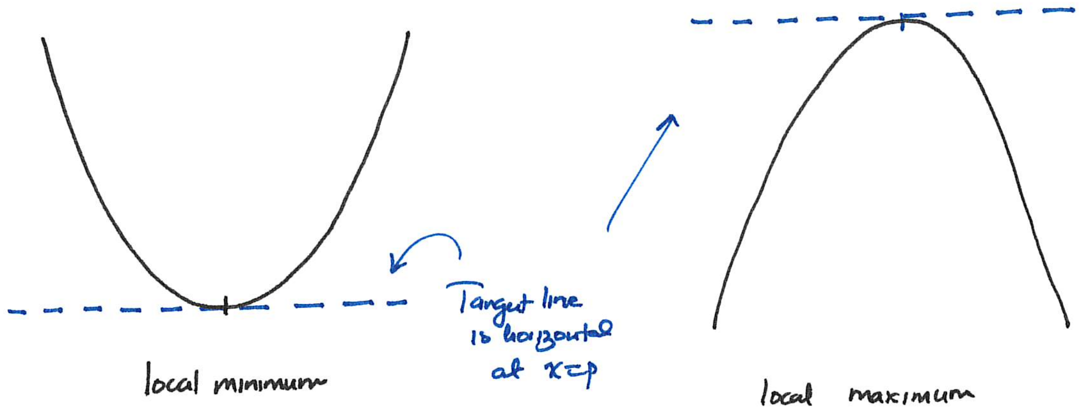
Local maxima and minima

Suppose p is a point in the domain of f

* f has a local minimum at p if $f(p)$ is less than or equal to values of f @ points near p

** f has a local maximum at p if $f(p)$ is greater than or equal to values of f @ points near p

Determining local maxima / minima



local minimum
@ $x=p$

local maximum
@ $x=p$

Critical point

For any function f, a point p where $f'(p)=0$ or $f'(p)$ is undefined is called a critical point.

i.e. If $f'(p)=0$, then p is a critical point

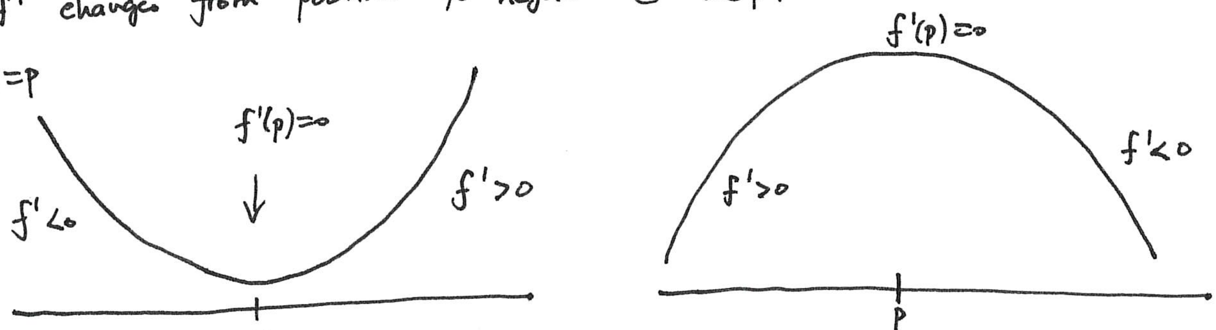
$f(p)$ is called a critical value.

> If f is continuous and has a local maximum or minimum at $x=p$, then p is a critical point

Test for local maxima or minima

First derivative Test

1. If f' changes from negative to positive @ $x=p$, then f has a local minimum @ $x=p$
2. If f' changes from positive to negative @ $x=p$, then f has a local maximum @ $x=p$



local minimum

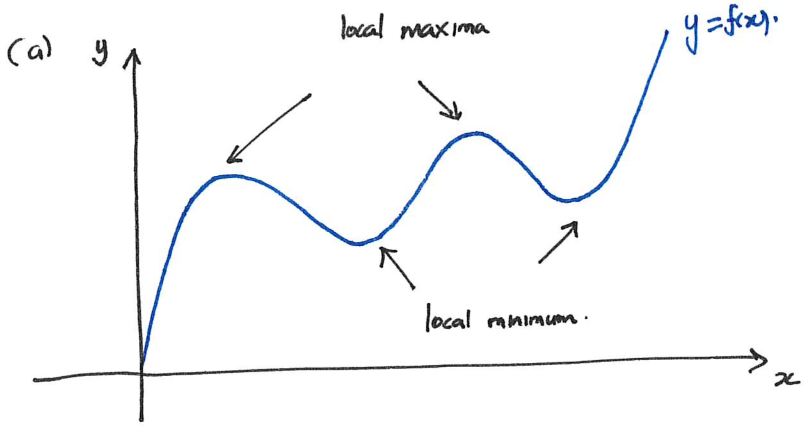
local maximum.

Second derivative Test

1. If $f''(p) > 0$, then f has a local minimum at $x = p$
2. If $f''(p) < 0$, then f has a local maximum at $x = p$.
3. If $f''(p) = 0$, then the test is inconclusive.

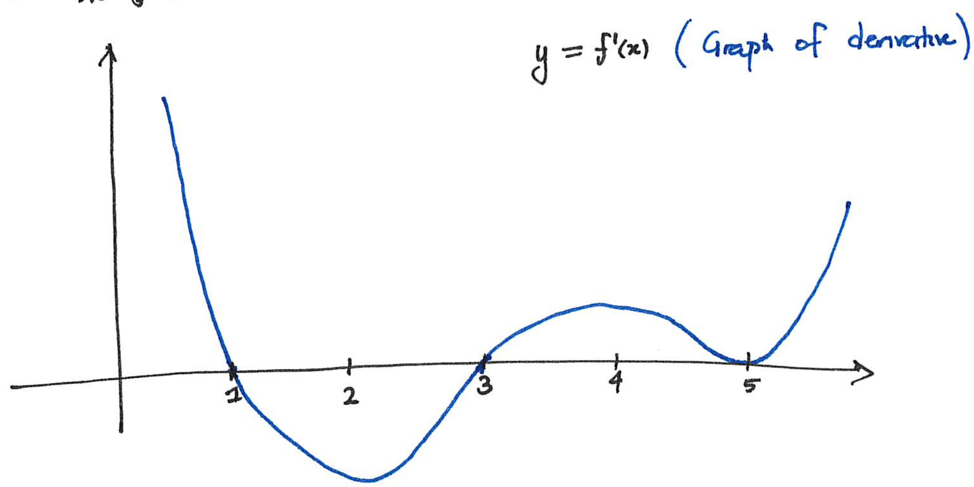
Examples.

(a)



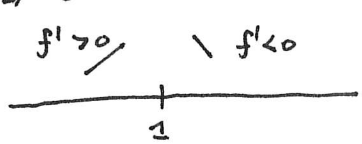
- f has 4 critical points

(b) ~~If $f(x) = 2x^4$~~

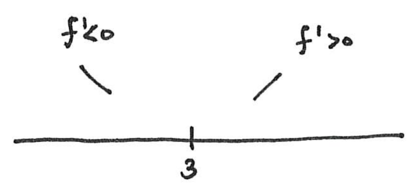


@ $x = 1$ (local maximum)

$f'(1) = 0$



@ $x = 3$ local minimum



@ $x = 5$

f' does not change sign so neither local max/min.