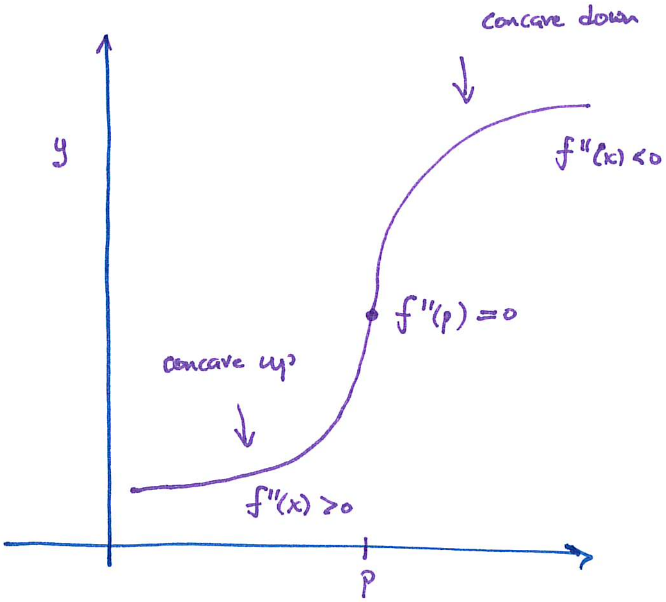
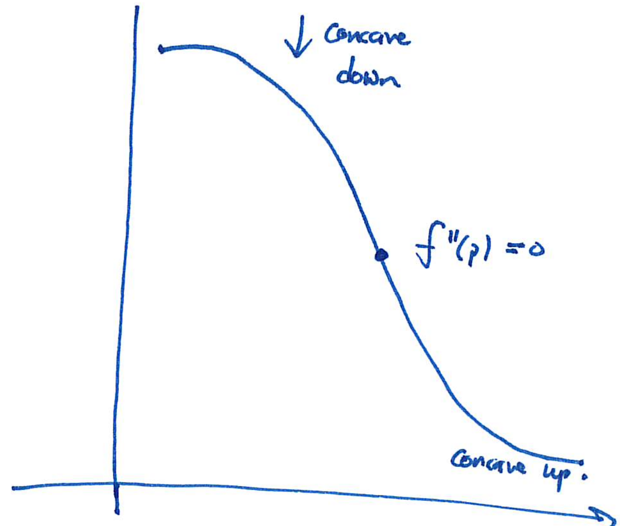
4.2 Inflection Points

A point at which the graph of a function f changes concavity is called an inflection point of f.





If f'' is zero or undefined at p, then f is a possible inflection point

To test whether p is an inflection point, $f''(x)$ must change sign.

Example

$$f(x) = 2x^3 + 3x^2 - 36x + 5.$$

① Critical points

$$f'(x) = 6x^2 + 6x - 36$$

$$6x^2 + 6x - 36 = 0$$

$$6(x^2 + x - 6) = 0 \Rightarrow 6(x^2 - 2x + 3x - 6) = 0$$

$$x(x-2) + 3(x-2) = 0 \Rightarrow 6(x-2)(x+3) = 0$$

$x = 2$ & $x = -3$ are critical points

② Classification

$$f''(x) = 12x + 6$$

$f''(2) = 12 \cdot 2 + 6 > 0 \Rightarrow x = 2$ is a local min

$f''(-3) = 12 \cdot (-3) + 6 < 0 \Rightarrow x = -3$ is a local max

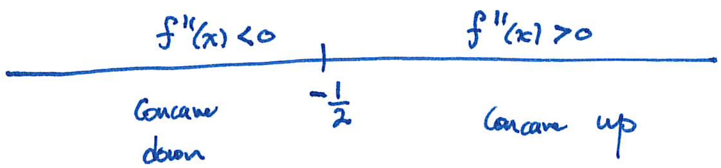
② Inflection point(s)

$f''(x) = 0 \Rightarrow 12x + 6 = 0 \Rightarrow 12x = -6$

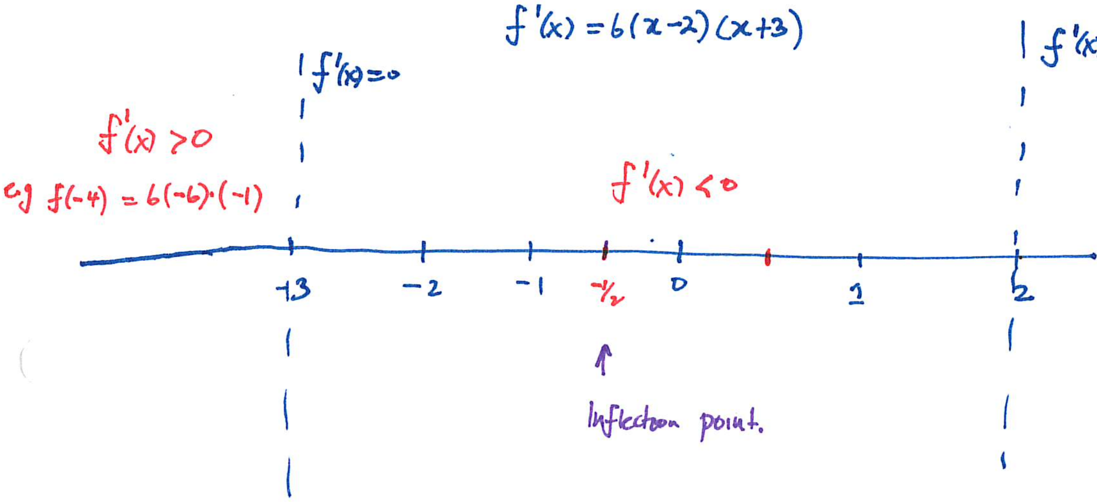
$x = -\frac{1}{2}$

Does $f''(x)$ change sign?

$f''(x) = 12(x + \frac{1}{2})$

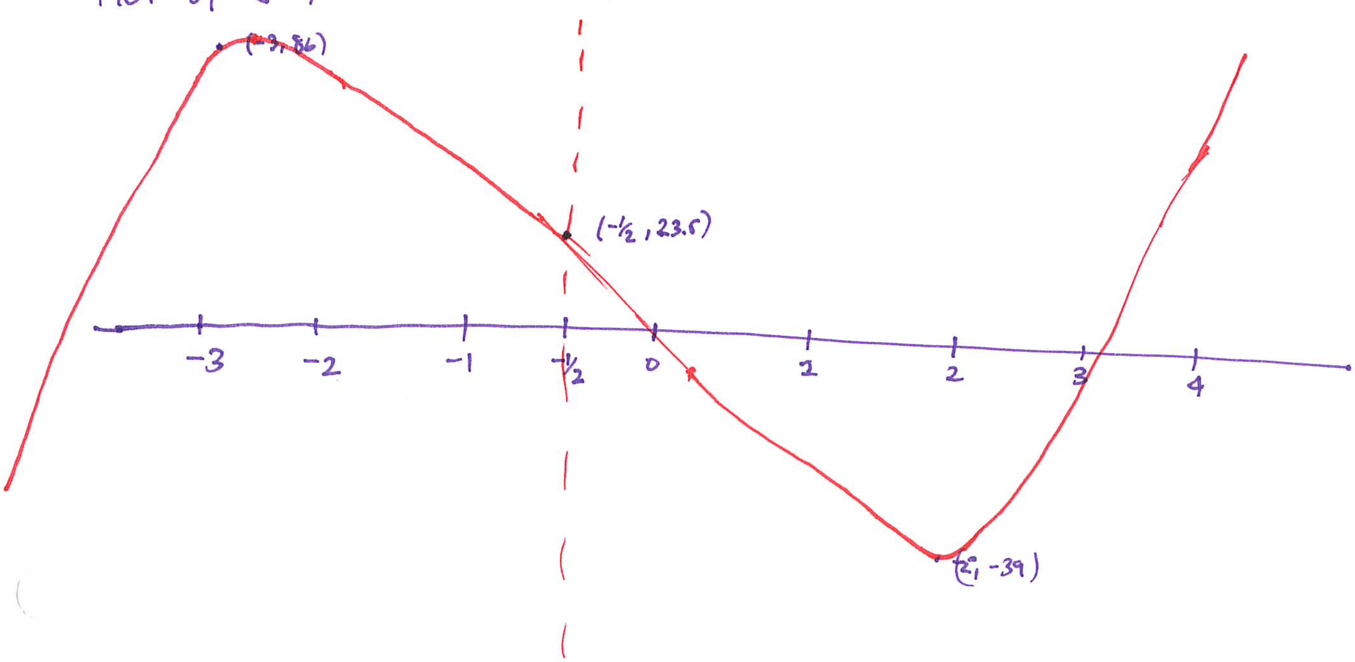


$f'(x) = 6(x-2)(x+3)$



- f is increasing on $(-\infty, -3)$
- f is decreasing on $(-3, 2)$
- f is increasing on $(2, \infty)$

Plot of $f(x) = 2x^3 + 3x^2 - 36x + 5$



Increasing and concave

do