Profit, Cost and Revenue

\[ \text{Profit} = \text{Revenue} - \text{Cost} \]

\[ \Pi(q) = R(q) - C(q) \]

Goal

Maximize profit.

\[ \$ \]

The ideal level of production is \( q_1 < q < q_2 \).

From our critical point analysis we know that the maximum occurs when

\[ \Pi'(q) = 0. \]

\[ \Pi'(q) = R'(q) - C'(q) \]

\[ = MR - MC \]

\[ @ \text{ The maximum point of production } \Pi'(q) = 0 \Rightarrow 0 = MR - MC. \]

\[ \Rightarrow MR = MC. \]

The maximum (or minimum) profit can occur when

Marginal profit = 0 i.e

Marginal revenue = Marginal cost

In fact to find the max/min of any function \( f \), we find the critical points i.e. \( f'(x) = 0 \) and solve and look for the local max or min.
Example #1

At the point $q_2$, $MC = MR$ so profit is maximized at $q_2$, between $q_3$ and $q_4$.

We can sketch the marginal revenue and marginal cost.

Profit function

$\pi'(q) = 0$
\[ a(q) = \frac{C(q)}{q} = \frac{q^3 - 12q^2 + 60q}{q} = q^2 - 12q + 60 \]

Find critical point.

\[ a'(q) = 2q - 12q \]

\[ 2q - 12q = 0 \implies q = 6. \]

Check that this is a minimum.

\[ a''(q) = 2 > 0 \text{ so } q = 6 \text{ is a local minimum.} \]

The marginal cost (from \( C(q) = q^3 - 12q^2 + 60q \)) is

\[ C'(q) = 3q^2 - 24q + 60 \]

\[ MC(6) = 24. \]

The average cost \( a(6) = 24 \)

Desmos plot of comparison.