Average cost

To turn a profit, the price of a product has to be above the average cost of production.

Average cost $= \dfrac{C(q)}{q}$ - cost per unit of producing a certain quantity.

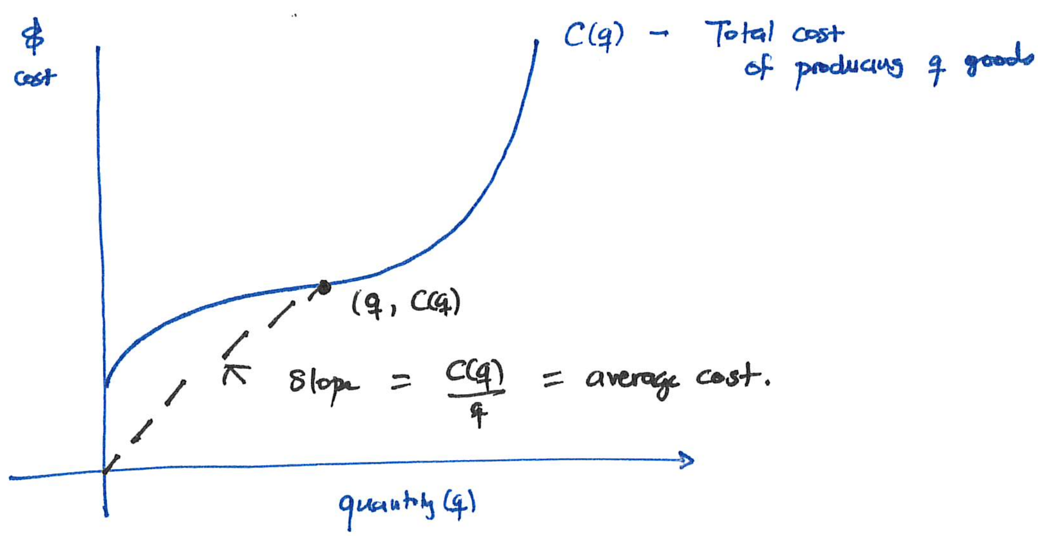
Units - dollars/item.

Example

If $C(q) = 0.01q^3 - 0.6q^2 + 13q + 1000$ (in dollars), then the average cost of producing 100 goods is

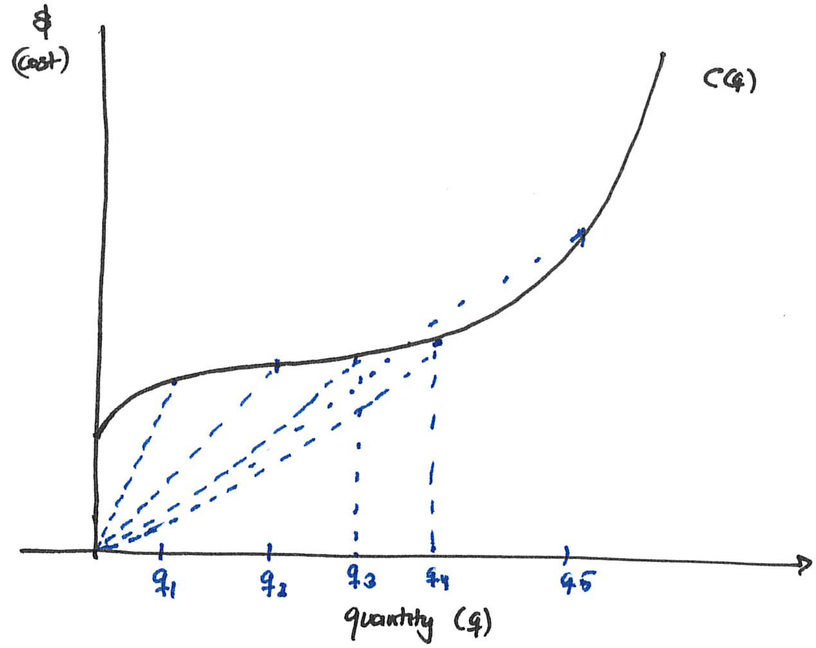
$$\frac{C(100)}{100} = \frac{\$6300}{100} = \$63/\text{item}.$$

Visualizing the average cost



Minimizing Average cost

1102



1. The average cost is large for small q (because the fixed costs have already been paid and are spread over fewer goods

2. There is a minimum value of average cost – It occurs when the average cost is tangent to the cost curve.

i.e At minimum average cost

Marginal Cost = ~~Marg~~ Average cost.

Summary

① If Marginal Cost < Average cost, increasing production decreases average cost

② If Marginal Cost > Average cost, increasing production increases average cost.

③ Marginal cost = Average cost at critical point of average cost.

Example

Let $C(q) = q^3 - 12q^2 + 60q$, $0 \leq q \leq 8$

Find the value of q that minimizes the average cost

Example #1

Revenue is given by $R(q) = 450q$ and cost $C(q) = 10{,}000 + 3q^2$.

At what quantity is profit maximized?

We want to find q such that $\pi'(q) = 0 \Rightarrow MR = MC$.

$MR(q) = 450$

$MC(q) = 6q$

$MR = MC \Rightarrow 450 = 6q \Rightarrow q = \frac{450}{6} = \underline{75 \text{ items}}$

Checking that $q = 75$ items is a maximum

$\pi(q) = 450q - (10{,}000 + 3q^2)$

$\pi'(q) = 450 - 6q$

$\pi''(q) = -6 < 0 \Rightarrow q = 75$ is a maximum.

Example #2

Maximizing Revenue (They do)

The demand equation for a product is $p = 45 - 0.01q$

(a) Write the Revenue as a function of q

(b) Find the quantity that maximizes revenue

(c) ~~What is the total revenue at~~

What is the price? What is the Revenue @ this price?

(a) Revenue = price x quantity sold

$= (45 - 0.01q)\, q = 45q - 0.01q^2$

(b) We find the critical point of the Revenue function

$R'(q) = 45 - 0.02q \Rightarrow q = \frac{45}{0.02} = \frac{4500}{2} = 2250$ items

$R''(q) = -0.02$ so $q = 2250$ items is indeed a maximum.

(c) $p = 45 - 0.01q \Rightarrow 45 - 0.01(2250) = 45 - 22.50 = \$22.50. \Rightarrow R = \$22.50 \cdot 2250$

$\$50{,}625.00$