Checklist and Pratice Exam
MATH 151 – SPRING 2019

1 Checklist

Functions

1. Interpretation of the derivative - *revenue, cost, profit functions, e.t.c*
2. Linear functions - *Constant rate of change; interpretation of slope, intercept; equations of lines*
3. Average rate of change and Relative rate of change
4. Marginal cost, revenue in the case of linear functions.
5. Break-even points of revenue and cost functions
6. Supply and Demand curves - *points of equilibrium*
7. Exponential functions - *constant % change; compounding interest; continuous growth*

Instantaneous Rate of change

1. Average velocity and Instantaneous velocity
2. Derivative as slope
3. Derivative function and interpretation
4. Using the derivative to approximate function values near tangent line - $f(x) \approx f(a) + f'(a)(x - a)$
5. Relative rate of change
6. Marginal values of cost, revenue, e.t.c - *Using the derivative to make decisions at the margins*

Formula for the Derivative

1. Know your formulas - Make a list of all of them and do at least one problem for each.

Application of the derivative

1. Local maxima and minima, critical points, inflection points
2. Global maxima and minima
3. Profit, Revenue and Cost -*maximizing profit*
4. Average cost function – *minimizing average cost*
5. Elasticity of demand
6. Impact of Elasticity on revenue

Accumulated Change

1. Estimate the change in a quantity from the rate of change of the quantity
2. $\int_a^b f(x)\,dx$ is the signed area under the graph of f.
3. Estimate the integral using the average of the left and right endpoint rules
4. Marginal cost and change in total cost.

2 Practice Exam

1. The annual revenue from McDonalds restaurants can be estimated by

$$R(t) = 19.1 + 1.8t$$

where $R(t)$ is the revenue in billions of dollars in year t, the time since January 1, 2010.

   (a) State the slope and y intercept of the revenue function, $R(t)$. Interpret each answer in terms of McDonald's revenue.

   (b) What is the estimated revenue on January 1, 2017?

   (c) Suppose instead, the revenue is estimated to grow at a rate of 3% per year (starting with \$19.1B in 2010). Write down a formula for the estimated revenue in year t.

   (d) Under this new growth rate, when will the revenue reach \$25B?

2. The demand and supply curves are given by

$$q = 100 - 2p \text{ and } q = 3p - 50,$$

respectively.

   (a) Explain the terms equilibrium price and quantity.

   (b) Find the equilibrium price and quantity.

   (c) A tax of \$5.00 is imposed on the suppliers. Find the new equilibrium price and quantity.

   (d) How much of the \$5.00 tax is paid by the consumers?

3. Suppose you deposit \$5,000 in a savings account at an interest rate of 5% compounded continuously. Meanwhile, your wealthy neighbor invests \$10,000 in an account with an interest rate of 3% compounded quarterly. When will the 2 accounts have the same balance?

4. The annual sales, in billions of dollars, of the Hershey cooperation are given by a function $S = f(t)$, where t is the time in years since January 1, 2015.

   (a) Interpret $f(8) = 5.1$ and $f'(8) = 0.22$ in terms of Hershey sales.Give the appropriate units in each case.

   (b) Estimate $f(10)$. Interpret your answer in terms of Hershey sales.

5. After investing \$1000 at a annual rate of 7% compounded continuously in 2010, the balance on the account is given by $B = f(t)$ where t is the time measured since 2010.

   (a) Find the function $B = f(t)$ that gives the balance on the account in year t.

   (b) Find $\frac{dB}{dt}$ in 2015; interpret your answer in terms of the account balance.

(c) Calculate the relative rate of change in 2015; interpret your answer in terms of the balance of the bank account.

6. Find the derivative of each of the following functions. If you apply a rule, leave your solution in the form of the rule. DO NOT simplify your solution

   (a) $y = \sqrt[3]{x} + 4x^2 + \dfrac{8}{x^4}$

   (b) $y = e^{x^2}$

   (c) $y = \dfrac{5x^2}{1+x^3}$

   (d) $y = x\ln(2x+1)$

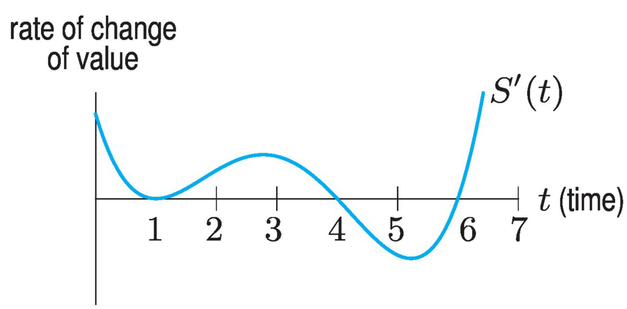
7. The quantity demanded of a certain product, q is given in terms of the price, p, by

$$q = 1000e^{-0.02p}$$

   (a) Write the revenue as a function of price.

   (b) Find the rate of change of revenue with respect to the price for $p = \$10$. Interpret your answer in economic terms.

8. The value of an investment at time t is given by $S(t)$. The rate of change, $S'(t)$, of the value of the investment is shown in the figure below.



   (a) What are the critical points of $S(t)$.

   (b) Identify each critical point as a local maximum, local minimum, or neither.

   (c) Explain the financial significance of each of the critical points.

9. For the function $f(x) = 2x^3 - 9x^2 + 12x + 1$.

   (a) Find and classify the critical points of f.

   (b) Find any inflection points of f.

   (c) Find the global maximum and minimum values of f on the interval $-0.5 \le x \le 3$

10. The demand equation for a quantity of product at a price of p is

$$p = -5q + 4,000$$

A company produces the product at a cost of $C = 5q + 5$.

   (a) Express the company's profit as a function of q.

(b) Find the production level that earns the largest profit.

(c) What is the largest possible profit?

11. The average cost per item of producing a product is given by

$$a(q) = 0.01q^2 - 0.6q + 13$$

(a) Find the production level that minimizes the average cost.

(b) What is the lowest average cost?

(c) What is the total cost of production $C(q)$.

(d) Find the production level that minimizes the *marginal cost*.

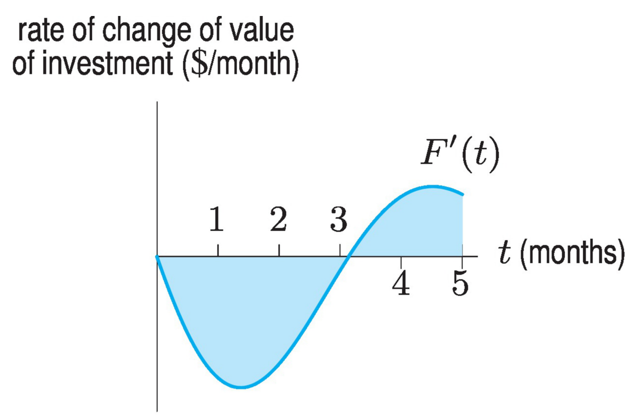
(e) Compute the marginal cost at $q = 30$. How does it compare to your solution in (b)? Explain this relationship.

12. The demand curve for a product is given by $q = 1000 - 2p^2$, where p is the price.

(a) Calculate the elasticity at $p = 15$.

(b) What should the seller do to increase revenue in this case?

13. The figure below shows $F'(t)$, the rate of change of the value, $F(t)$, of an investment over a 5-month period.



(a) When is the value of the investment increasing in value and when is it decreasing?

(b) Does the investment increase or decrease in value during the 5 months.

14. The marginal cost $C'(q)$ (in dollars per unit) of producing q units is given in the following table.

q	0	100	200	300	400	500	600
$C'(q)$	25	20	18	22	28	35	45

(a) If the fixed cost is $10,000, estimate the total cost of producing 500 units.

(b) How much would the total cost increase if production were increased by one unit, to 501 units.