1 Practice Exam solutions

1. The annual revenue from McDonalds restaurants can be estimated by

\[ R(t) = 19.1 + 1.8t \]

where \( R(t) \) is the revenue in billions of dollars in year \( t \), the time since January 1, 2010.

(a) State the slope and \( y \) intercept of the revenue function, \( R(t) \). Interpret each answer in terms of McDonald’s revenue.

\[-intercept is $19.1 corresponds to revenue on Jan 1, 2010
\[-slope is $1.8 billion dollars per year is the rate of increase of Revenues \]

(b) What is the estimated revenue on January 1, 2017?

$31.7 Billion

(c) Suppose instead, the revenue is estimated to grow at a rate of 3% per year (starting with $19.1B in 2010). Write down a formula for the estimated revenue in year \( t \).

\[ R(t) = 19.1(1.03)^t \]

(d) Under this new growth rate, when will the revenue reach $25B?

9.106 years

2. The demand and supply curves are given by

\[ q = 100 - 2p \] and \[ q = 3p - 50, \]

respectively.

(a) Explain the terms equilibrium price and quantity.

This is the price and quantity such that SUPPLY = DEMAND

(b) Find the equilibrium price and quantity.

\[ p = $30, q = 40 \] items

(c) A tax of $5.00 is imposed on the suppliers. Find the new equilibrium price and quantity.

The new supply equation is \( 3(p - 5) - 50, \) we can use that along with the old demand equation to get \( p = 33, q = 34 \) items

(d) How much of the $5.00 tax is paid by the consumers?

The price goes up by $3, so the consumer has picked up 3 of the 5 dollars in tax

3. Suppose you deposit $5,000 in a savings account at an interest rate of 5% compounded continuously. Meanwhile, your wealthy neighbor invests $10,000 in an account with an interest rate of 3% compounded quarterly. When will the 2 accounts have the same balance?

Set up the 2 equations as

\[ 500e^{0.05t} = 10,000(1 + \frac{0.03}{4})^{4t} \]

and solve for \( t \) to get \( t = 34.46 \). Use the laws of logs carefully!
4. The annual sales, in billions of dollars, of the Hershey cooperation are given by a function $S = f(t)$, where $t$ is the time in years since January 1, 2015.

(a) Interpret $f(8) = 5.1$ and $f'(8) = 0.22$ in terms of Hershey sales. Give the appropriate units in each case.

(a) In 2023, the projected sales are $5.1$ billion
(b) In 2023, sales are increasing at a rate of $0.22$ billion per year

(b) Estimate $f(10)$. Interpret your answer in terms of Hershey sales.

5. After investing $1000$ at a annual rate of $7\%$ compounded continuously in 2010, the balance on the account is given by $B = f(t)$ where $t$ is the time measured since 2010.

(a) Find the function $B = f(t)$ that gives the balance on the account in year $t$.

$B = 1000e^{0.07t}$

(b) Find $\frac{dB}{dt}$ in 2015; interpret your answer in terms of the account balance.

$99.33$ dollars per year, this is the rate at which the account is growing in 2015.

(c) Calculate the relative rate of change in 2015; interpret your answer in terms of the balance of the bank account.

$0.07$, the account is growing at $7\%$ per year

6. Find the derivative of each of the following functions. If you apply a rule, leave your solution in the form of the rule. DO NOT simplify your solution

(a) $y = \sqrt[3]{x} + 4x^2 + \frac{8}{x^4}$

$b) y = e^{x^2} \cdot 2x e^{2x}$

(c) $y = \frac{5x^2}{1 + x^3}$

$\frac{(1 + x^3)(10x) - (5x^2)(3x^2)}{(1 + x^3)^2}$

(d) $y = x \ln(2x + 1)$

$1 \cdot \ln(2x + 1) + x \cdot \frac{2}{2x + 1}$

7. The quantity demanded of a certain product, $q$, is given in terms of the price, $p$, by

$q = 1000e^{-0.02p}$

(a) Write the revenue as a function of price.

Revenue = Price $\times$ Quantity so $R(p) = p \cdot 1000e^{-0.02p}$

(b) Find the rate of change of revenue with respect to the price for $p = 10$. Interpret your answer in economic terms.

$R'(10) = 654.98$ so increasing the price by $1.00$ to $11$ will result in $654.98$ additional revenue.
8. The value of an investment at time $t$ is given by $S(t)$. The rate of change, $S'(t)$, of the value of the investment is shown in the figure below.

(a) What are the critical points of $S(t)$.
1, 4, 6
(b) Identify each critical point as a local maximum, local minimum, or neither.
$t = 1$ is neither because the slope does not change sign, $t = 4$ is a local max, and $t = 6$ is a local min
(c) Explain the financial significance of each of the critical points.
At time $t = 1$ the investment momentarily stopped increasing in value, though it started increasing again immediately afterward. At $t = 4$, the value peaked and began to decline. At $t = 6$, it started increasing again.

9. For the function $f(x) = 2x^3 - 9x^2 + 12x + 1$.
(a) Find and classify the critical points of $f$.
$x = 1$ (local max) $x = 2$ (local min).
(b) Find any inflection points of $f$.
$x = 1.5$ inflection point
(c) Find the global maximum and minimum values of $f$ on the interval $-0.5 \leq x \leq 3$
$\text{global max is 10, global min is } -7.5$.

10. The demand equation for a quantity of product at a price of $p$ is

\[ p = -5q + 4,000 \]

A company produces the product at a cost of $C = 5q + 5$.

(a) Express the company’s profit as a function of $q$.

\[ \pi(q) = (-5q + 4000)q - (5q + 5) \]
(b) Find the production level that earns the largest profit.

\[ q = 399.5 \approx 400 \]
(c) What is the largest possible profit?
\( \pi(400) = \$797995 \).

11. The average cost per item of producing a product is given by

\[ a(q) = 0.01q^2 - 0.6q + 13 \]

(a) Find the production level that minimizes the average cost.
\( q = 30 \).

(b) What is the lowest average cost?
\( \$4/\text{item} \)

(c) What is the total cost of production \( C(q) \).
\[ C(q) = (0.01q^2 + 0.6q + 13)q \]

(d) Find the production level that minimizes the marginal cost.
The Marginal cost, \( MC(q) = 0.03q^2 - 1.2q + 13 \) so to find the production level that minimized \( MC(q) \) we need the critical points of \( MC(q) \). Find \( MC'(q) \) at set that equal to zero to get \( q = 20 \). Check that this is a minimum.

(e) Compute the marginal cost at \( q = 30 \). How does it compare to your solution in (b)? Explain this relationship. \( MC(20) = \$4/\text{item} \) This is the same as our answer from \( b \) because at minimum average cost, the average cost is equal to the marginal cost.

12. The demand curve for a product is given by \( q = 1000 - 2p^2 \), where \( p \) is the price.

(a) Calculate the elasticity at \( p = 15 \). \( E = 1.636 \).

(b) What should the seller do to increase revenue in this case?
\( E > 1 \) so decreasing prices increases revenues.

13. The figure below shows \( F'(t) \), the rate of change of the value, \( F(t) \), of an investment over a 5-month period.

(a) When is the value of the investment increasing in value and when is it decreasing?
The investment decreased in value during the first 3 months, since the rate of change of value is negative then. The value rose during the last 2 months.

(b) Does the investment increase or decrease in value during the 5 months.
Total change in value = \( \int_0^5 F'(t) \, dt \). This is the area under the curve. Since the area below the \( x \)-axis is greater than the area above the \( x \)-axis the integral is negative. Thus, the value of the investment during this time period has decreased.
14. The marginal cost $C'(q)$ (in dollars per unit) of producing $q$ units is given in the following table.

<table>
<thead>
<tr>
<th>$q$</th>
<th>0</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C'(q)$</td>
<td>25</td>
<td>20</td>
<td>18</td>
<td>22</td>
<td>28</td>
<td>35</td>
<td>45</td>
</tr>
</tbody>
</table>

(a) If the fixed cost is $10,000, estimate the total cost of producing 500 units.

$21,800$

(b) How much would the total cost increase if production were increased by one unit, to 501 units.

From the table, the marginal cost is $35/item.