Checklist and Pratice Exam solutions
MATH 151 – SPRING 2019

1 Practice Exam solutions

1. The annual revenue from McDonalds restaurants can be estimated by

$$R(t) = 19.1 + 1.8t$$

where $R(t)$ is the revenue in billions of dollars in year t, the time since January 1, 2010.

   (a) State the slope and y intercept of the revenue function, $R(t)$. Interpret each answer in terms of McDonald's revenue.
   y-intercept is \$19.1 corresponds to revenue on Jan 1, 2010
   slope is \$1.8 billion dollars per year is the rate of increase of Revenues

   (b) What is the estimated revenue on January 1, 2017?
   \$31.7 Billion

   (c) Suppose instead, the revenue is estimated to grow at a rate of 3% per year (starting with \$19.1B in 2010). Write down a formula for the estimated revenue in year t.
   $R(t) = 19.1(1.03)^t$

   (d) Under this new growth rate, when will the revenue reach \$25B?
   9.106 years

2. The demand and supply curves are given by

$$q = 100 - 2p \text{ and } q = 3p - 50,$$

respectively.

   (a) Explain the terms equilibrium price and quantity.
   This is the price and quantity such that SUPPLY = DEMAND

   (b) Find the equilibrium price and quantity.
   $p = \$30, q = 40$ items

   (c) A tax of \$5.00 is imposed on the suppliers. Find the new equilibrium price and quantity.
   The new supply equation is $3(p - 5) - 50$, we can use that along with the *old* demand equation to get $p = 33, q = 34$ items

   (d) How much of the \$5.00 tax is paid by the consumers?
   The price goes up by \$3, so the consumer has picked up 3 of the 5 dollars in tax

3. Suppose you deposit \$5,000 in a savings account at an interest rate of 5% compounded continuously. Meanwhile, your wealthy neighbor invests \$10,000 in an account with an interest rate of 3% compounded quarterly. When will the 2 accounts have the same balance?
Set up the 2 equations as

$$500e^{0.05t} = 10,000\left(1 + \frac{0.03}{4}\right)^{4t}$$

and solve for t to get $t = 34.46$. Use the laws of logs carefully!

4. The annual sales, in billions of dollars, of the Hershey cooperation are given by a function $S = f(t)$, where t is the time in years since January 1, 2015.

   (a) Interpret $f(8) = 5.1$ and $f'(8) = 0.22$ in terms of Hershey sales.Give the appropriate units in each case.
   (a) In 2023, the projected sales are \$5.1 billion   (b) In 2023, sales are increasing at a rate of \$0.22 billion per year

   (b) Estimate $f(10)$. Interpret your answer in terms of Hershey sales.
   5.54 billion dollars

5. After investing \$1000 at a annual rate of 7% compounded continuously in 2010, the balance on the account is given by $B = f(t)$ where t is the time measured since 2010.

   (a) Find the function $B = f(t)$ that gives the balance on the account in year t.
   $B = 1000e^{0.07t}$

   (b) Find $\frac{dB}{dt}$ in 2015; interpret your answer in terms of the account balance.
   \$99.33 dollars per year, this is the rate at which the account is growing in 2015.

   (c) Calculate the relative rate of change in 2015; interpret your answer in terms of the balance of the bank account.
   0.07, the account is growing at 7% per year

6. Find the derivative of each of the following functions. If you apply a rule, leave your solution in the form of the rule. DO NOT simplify your solution

   (a) $y = \sqrt[3]{x} + 4x^2 + \frac{8}{x^4}$
   $\frac{1}{3}x^{-\frac{2}{3}} + 8x - 32x^{-5}$

   (b) $y = e^{x^2}$ $2xe^{x^2}$

   (c) $y = \frac{5x^2}{1 + x^3}$

$$\frac{(1 + x^3)(10x) - (5x^2)(3x^2)}{(1 + x^3)^2}$$

   (d) $y = x \ln(2x + 1)$

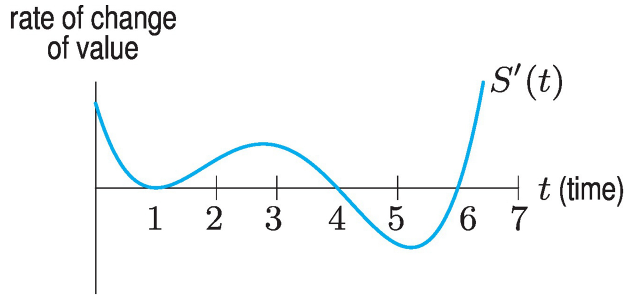
$$1 \cdot \ln(2x + 1) + x \cdot \frac{2}{2x + 1}$$

7. The quantity demanded of a certain product, q is given in terms of the price, p, by

$$q = 1000e^{-0.02p}$$

   (a) Write the revenue as a function of price.
   Revenue = Price $\times$ Quantity so $R(p) = p \cdot 1000e^{-0.02p}$

   (b) Find the rate of change of revenue with respect to the price for $p = \$10$. Interpret your answer in economic terms.
   $R'(10) = \$654.98$ so increasing the price by \$1.00 to \$11 will result in \$654.98 additional revenue.



8. The value of an investment at time t is given by $S(t)$. The rate of change, $S'(t)$, of the value of the investment is shown in the figure below.

   (a) What are the critical points of $S(t)$.
   $1, 4, 6$

   (b) Identify each critical point as a local maximum, local minimum, or neither.
   $t = 1$ is neither because the slope does not change sign, $t = 4$ is a local max, and $t = 6$ is a local min

   (c) Explain the financial significance of each of the critical points.
   At time $t = 1$ the investment momentarily stopped increasing in value, though it started increasing again immediately afterward. At $t = 4$, the value peaked and began to decline. At $t = 6$, it started increasing again.

9. For the function $f(x) = 2x^3 - 9x^2 + 12x + 1$.

   (a) Find and classify the critical points of f.
   $x = 1$ (local max) $\quad x = 2$ (local min).

   (b) Find any inflection points of f.
   $x = 1.5$ inflection point

   (c) Find the global maximum and minimum values of f on the interval
   $-0.5 \leq x \leq 3$
   global max is 10, global min is -7.5.

10. The demand equation for a quantity of product at a price of p is

$$p = -5q + 4,000$$

A company produces the product at a cost of $C = 5q + 5$.

   (a) Express the company's profit as a function of q.

$$\pi(q) = (-5q + 4000)q - (5q + 5)$$

   (b) Find the production level that earns the largest profit.

$$q = 399.5 \approx 400$$

(c) What is the largest possible profit?
$\pi(400) = \$797995$.

11. The average cost per item of producing a product is given by

$$a(q) = 0.01q^2 - 0.6q + 13$$

(a) Find the production level that minimizes the average cost.
$q = 30$.

(b) What is the lowest average cost?
$\$4/item$

(c) What is the total cost of production $C(q)$.
$C(q) = (0.01q^2 + 0.6q + 13)q$

(d) Find the production level that minimizes the *marginal cost*.
The Marginal cost, $MC(q) = 0.03q^2 - 1.2q + 13$ so to find the production level that minimized $MC(q)$ we need the critical points of $MC(q)$. Find $MC'(q)$ at set that equal to zero to get $q = 20$. Check that this is a minimum.

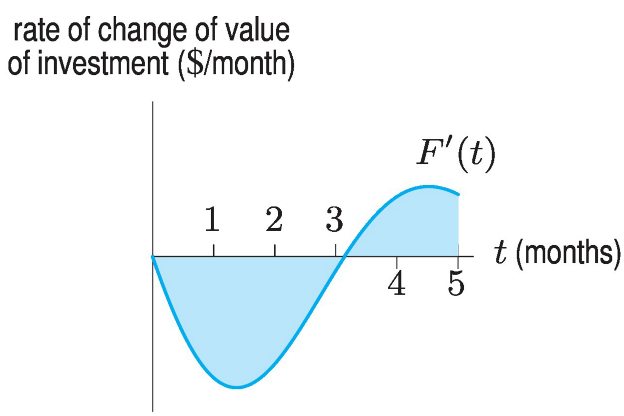
(e) Compute the marginal cost at $q = 30$. How does it compare to your solution in (b)? Explain this relationship. $MC(20) = \$4/item$ This is the same as our answer from b because at minimum average cost, the average cost is equal to the marginal cost.

12. The demand curve for a product is given by $q = 1000 - 2p^2$, where p is the price.

(a) Calculate the elasticity at $p = 15$. $E = 1.636$.

(b) What should the seller do to increase revenue in this case?
$E > 1$ so decreasing prices increases revenues.

13. The figure below shows $F'(t)$, the rate of change of the value, $F(t)$, of an investment over a 5-month period.



(a) When is the value of the investment increasing in value and when is it decreasing?
The investment decreased in value during the first 3 months, since the rate of change of value is negative then. The value rose during the last 2 months.

(b) Does the investment increase or decrease in value during the 5 months.
Total change in value $= \int_0^5 F'(t)\,dt.$. This is the area under the curve. Since the area below the x-axis is greater than the area above the x-axis the integral is negative. Thus, the value of the investment during this time period has decreased.

14. The marginal cost $C'(q)$ (in dollars per unit) of producing q units is given in the following table.

q	0	100	200	300	400	500	600
$C'(q)$	25	20	18	22	28	35	45

(a) If the fixed cost is $\$10,000$, estimate the total cost of producing 500 units.
$\$21,800$

(b) How much would the total cost increase if production were increased by one unit, to 501 units.
From the table, the marginal cost is $\$35/item$.