

MA 302 – Spring 2020
Homework 2: Due on Wednesday, February 5

Exercise 1: Matrix and Array manipulations

1. Let $\text{vec} = [4 \ 5 \ 9 \ 6]$
 - (a) Subtract 3 from each element.
 - (b) Add 11 to the odd-index elements
 - (c) Compute the square root of each element
 - (d) Raise each element to the power of 3.
2. Create a vector \mathbf{v} with elements $v_n = \frac{(-1)^n}{2n-1}$ for $n = 1, 2, 3, \dots, N$ for any N . Find the sum of the 100-element vector.

Hints:

- start with the vector of all -1 s and perform the appropriate element-wise operations
 - Use the MATLAB `sum` command
3. Given a vector \mathbf{t} , write down MATLAB expressions that will compute (componentwise)
 - (a) $\ln(2 + t + t^2)$
 - (b) $\frac{1}{1 + t^2}$

Test your code with any vector \mathbf{t} of your choice.

4. Recall that the angle between non-zero vectors \mathbf{u} and \mathbf{v} is given by

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}$$

where $\|\mathbf{v}\|$ is the Euclidean *norm* of the vector \mathbf{v} . Use Matlab to calculate the angle between $\langle 2, 2, -1 \rangle$ and $\langle 5, -3, 2 \rangle$.

5. Given the vectors $\mathbf{x} = [1 \ 3 \ 7]$ and $\mathbf{y} = [2 \ 4 \ 2]$ and matrices $\mathbf{A} = \begin{bmatrix} 3 & 1 & 6 \\ 5 & 2 & 7 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 & 4 \\ 7 & 8 \\ 2 & 2 \end{bmatrix}$, determine which of the following statements can be correctly executed in MATLAB. In each case explain *briefly* what the operation does or why MATLAB yields an error message.

- $\mathbf{x} + \mathbf{y}$
- $\mathbf{x} + \mathbf{A}$
- $\mathbf{x}' + \mathbf{y}$
- $\mathbf{A} - [\mathbf{x}' \ \mathbf{y}']$
- $[\mathbf{x}; \ \mathbf{y}']$
- $\mathbf{B} * \mathbf{A}$
- $\mathbf{A} .* \mathbf{B}$
- $\mathbf{A}' .* \mathbf{B}$
- $\mathbf{B} ./ \mathbf{x}'$

- `ones(1,3)*A`
 - `4./B`
 - `[A, fliplr(A)]`
 - `B(3,4) = 2;`
6. The MATLAB `\` operator can be used to solve linear systems. Let A be a random 5×5 matrix and b be a random 5×1 vector. Given $Ax = b$ find x by calling `x = A\b`, compare your result to using MATLAB's `inv` operator. Having found x , check whether $Ax - b$ is close to zero.
7. The MATLAB `magic` function returns a square matrix whose row sums, column sums and the sums of the two diagonals are the same. Consider the matrix

```
1           A = magic(201); % do not print the entries
```

The matrix A has 201 row sums and 201 column sums, and two diagonal sums. These 404 sums should be exactly the same, and you **could** verify this by printing them and checking visually but it is easy to miss a digit with so many numbers. **Instead**, verify that A is a magic square by constructing 201 column sums (without printing them) and computing the maximum and minimum values of the column sums. Do the same for the 201 row sums, and compute the two diagonal sums. Check that the six values are the same. If the maximum and minimum values are the same, then this confirms that the row, diagonal and column sums are the same.

Hints:

- Use the MATLAB functions `min` and `max`
 - Use `>>help sum` to determine the result of calling `sum(X)` where X is a matrix or vector.
 - The Matlab `diag` function extracts the diagonal of a matrix, you can extract the other diagonal by calling `diag(fliplr(A))`
8. In the following exercise, use the provided `loyola.jpg` or any colorful image of your choice. Define a 3D matrix `Img` storing the RGB values and view the image you have loaded as follows:

```
1           Img = imread('loyola.jpg');
2           imshow(Img);
3           %enable viewing of RGB values
4           datacursormode('on')
```

Notice that if you click on your image, you can view the corresponding red, blue and green values at each position.

- (a) Confirm that `Img` is a 3D matrix
- (b) Create a new image (`ex1_prob8b.jpg`) that consists of the original image and its reflection as shown in Figure 1.
- (c) Loyola's official green color has RGB values 0, 104 and 87. Create a framed image (`ex1_prob8c.jpg`) that is 10 pixels wide in each direction by overriding the image at the boundary as shown in Figure 2.



Figure 1: Composite Image



Figure 2: Framed Image

- (d) Convert the color image to grey-scale by manipulating the 3D matrix as follows
- Create a new 2D matrix `BW1` that consists of the average of the RGB values at each point. Use `imshow(BW1)` to view your new image, save it as `ex1_prob8d_bw1.jpg`.
 - Create a new 2D matrix `BW2` using a weighted average of the RGB values as follows
$$BW2 = 0.21 R + 0.72 G + 0.07 B$$
Save your image as `ex1_prob8d_bw2.jpg`.
 - Finally create `BW3` by averaging the maximum and minimum values at each point as
$$BW3 = (\min(R,G,B) + \max(R,G,B)) / 2$$
and save your image as `ex1_prob8d_bw3.jpg`.
- (e) Which black and white image do you prefer? Explain.

Exercise 2: Plotting

For each of these plots label your jpg file as `ex2_probN_part.jpg` if the problem has multiple parts.

All your plots should have a title and all axes must be labelled

1. Look up the MATLAB function `polarplot` and use it to plot the polar curve $r = 1 + \sin(8\theta)$ for $0 \leq \theta \leq 2\pi$.
2. Consider the function $f(x) = \frac{1}{3}x^4 - 8x^2 + 4x + 1$ for $-10 \leq x \leq 10$.
 - (a) Find the first and second derivatives of f and include them in your summary file.
 - (b) Plot $y = f(x), y = f'(x), y = f''(x)$ on the same graph. Make sure to include a title and descriptive legend and use different colors and line styles for each function.
3. Plot the following 3D surfaces, for each specify the problem number and part
 - (a) $z = x^3 - 3xy^2, x, y \in [-15, 15] \times [-15, 15]$ Use `mesh`
 - (b) $z = \frac{\sin(x^2 + y^2)}{x^2 + y^2}$ on $x, y \in [-5, 5] \times [-5, 5]$. Use `surf` and let the z-axis go from -2 to 2 . **Note:** you can change the z-axis by specifying the limits as `zlim([-2 2])`.
4. Plot the level curves of $f(x, y) = -150xye^{-x^2-y^2}$ on $[-4, 4]^2$ using `contour(X,Y,Z)` where `X,Y` are matrices consisting of the x and y coordinates and function values in the matrix Z
5. Plot paraboloid $z = 2x^2 + y^2$ and the tangent plane $z = 4x + 2y - 3$ on $[-4, 4]^2$ on the same graph. Note: you can modify the edge of face color by specifying options for `'FaceColor'` and `'EdgeColor'` in your calls to `surf` and make sure to use the option `axis equal` to ensure the correct perspective.
6. Pick your favorite function f . Use `subplot` to illustrate on a 2×2 grid, the function f and 3 transformations of your choice.

Submission of exercises

Place all your files (`m-files`, `summary.txt`, `jpeg files`) in a folder named `lastname_hwN` and zip the folder to create a file `lastname_hwN.zip`. Email your zip file `lastname_hwN.zip` to `pchidyagwai@loyola.edu` with subject `MA302_hwN`.