MA 304: Homework 1 (due Friday January 24, 2020)

Reading

Sections 1.1, 1.2, 1.3

1. The population of deer living in some woods can be assumed to be growing according to the ODE

$$\frac{dP}{dt} = 0.4P\left(1 - \frac{P}{100}\right) - H,$$

with $P(t)$ equal to the number of deer at time t years after the initial time and the constant H equal to the number of deer harvested per year.

   (a) For values $H = 0, 6$ and 12 determine the equlibrium solutions of the ODE

   (b) Use the `dirfield.m` provided on the course website to plot the direction fields for the 3 values of H above for $0 \leq t \leq 25$ and $0 \leq P \leq 100$.

   (c) Include 5 integral curves with each plot for populations starting at $5, 20, 60, 80, 100$.

   (d) For each case, submit a printout of your plot of direction fields and integral curves and comment on the growth of the dear population.

2. Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between the temperature of the object, u and the ambient temperature, T. This means that

$$\frac{du}{dt} = -k(u - T)$$

where k is a positive constant. Suppose that the initial temperature at $t = 0$ is u_0, i.e $u(0) = u_0$. Find the temperature of the object at any time t.

3. For each of the following, verify that the given function is a solution of the differential equation

   (a) $t^2y'' + 5ty' + 4y = 0, t > 0; \quad y_1(t) = t^{-2}\ln(t)$

   (b) $y' - 2ty = 1; \quad y_1(t) = e^{t^2}\int_0^t e^{-s^2}\,ds + e^{t^2}$

   Hint: plug in $y_1(t)$ into the ODE and show that equality is satisfied

4. Some calc 2 review, do the following integrals

   (a) $\int e^x \sin(2x)\,dx$

   (b) $\int \frac{1-x}{x^2-4}\,dx$

   (c) $\int xe^{x^2}\,dx$