MA 304: Homework 3 (due Friday January 7, 2020)

Reading

Modeling with ODEs - Sections 2.3

1. **CSI?**
   At 7AM one morning, detectives find a murder victim in a closed meat locker. The temperature of the victim measures 88°. Assume the meat locker is always kept at 40°, and that at the time of death the victim's temperature was 98.6° and when the body is finally removed at 8AM, its temperature is 86°. Assuming Newton's law of cooling, when did the murder occur?

2. **A new fish**
   A fish tank initially contains $150l$ of water with $20g$ of salt dissolved in it. The concentration of salt in the tank needs to be increased from $\frac{20}{150}g/l$ to $1g/l$ to accommodate a new fish. Water containing $3g$ of salt per litre is allowed to flow into the tank at a rate of $2l$ per minute. The throughly stirred mixture is allowed to flow out at the same rate. Suppose the rate of inflow is increased to $2.5l/min$ but the outflow cannot be increased. The tank holds a maximum of 160 litres. Will the salt concentration reach the desired level before the container overflows?
   Hint: Show that the quantity of salt in the tank at time is

$$Q(t) = 1.5(300 + t) - \frac{430 \cdot 300^4}{(300 + t)^4}$$

3. **Mixing with 2 tanks**
   Assuming that at time $t = 0$, fresh water is pumped at a rate of $3gal/min$ into a 60 gallon tank initially filled with $q_0 \quad grams$ of dissolved salt. The resulting less-and-less salty mixture flows out at the same rate into a second 60 gallon tank that initially filled with pure water and from there it eventually spills to the ground. Assuming perfect mixing in both tanks, when will water in the second tank taste saltiest? How salty will it then be compared with the original salt mixture.

4. **Population doubling time**
   Section 2.3, page 48 - problem 10.