## MA 304: Homework 4 (due Monday February 17, 2020)

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## Reading

Sections 2.4, 2,5, 2.6

1. For each of the following determine whether the initial value problem has a unique solution. To receive any credit, fully justify your answer using existence and uniqueness facts covered in class.
   (a) $t^2y' + y = \sin(t), \quad y(1) = 0$
   (b) $\dfrac{dy}{dx} = \dfrac{x^2+x+1}{y-1}, \quad y(0) = -1;$
   (c) $y' = \sqrt{y}, \quad y(0) = 0$

2. Solve
$$y' = xy^{-2} - 2y$$
by recognizing it as an example of a Bernoulli equation.

3. Solve
$$\frac{dy}{dt} + Q(t)y = t$$
where
$$Q(t) = \begin{cases} 1, & 0 \le t \le 2, \\ 3, & t > 2 \end{cases}$$
with initial condition $y(0) = 1$

4. Suppose the rate of growth of population of fish in Loch Raven reservoir is given by
$$\frac{dF}{dt} = \frac{1}{2}F(4-F) - H$$
where $F(t)$ is the fish population at time $t$ in thousands and $t$ is time in years and $H$ is the number of fish harvested (in thousands) per year.
   (a) If no fish are harvested, how many fish can the reservoir support?
   (b) For $H = 1$, find and classify the equilibrium solutions. Show your solutions on a phase line.
   (c) If $H = 1$ and $F(0) = 0.5$? What happens to the fish population over time?
   (d) What is the bifurcation point for this fish growth model?

5. Problem 7, page 75

6. Solve
$$\frac{dy}{dx} = -\frac{2x^2+y}{x^2y-x}$$