Reading

Sections 2.4, 2.5, 2.6

1. For each of the following determine whether the initial value problem has a unique solution. To receive any credit, fully justify your answer using existence and uniqueness facts covered in class.
   (a) \( t^2 y' + y = \sin(t), \quad y(1) = 0 \)
   (b) \( \frac{dy}{dx} = \frac{x^2 + x + 1}{y - 1}, \quad y(0) = -1 \)
   (c) \( y' = \sqrt{y}, \quad y(0) = 0 \)

2. Solve
   \[ y' = xy^2 - 2y \]
   by recognizing it as an example of a Bernoulli equation.

3. Solve
   \[ \frac{dy}{dt} + Q(t)y = t \]
   where
   \[ Q(t) = \begin{cases} 1, & 0 \leq t \leq 2, \\ 3, & t > 2 \end{cases} \]
   with initial condition \( y(0) = 1 \)

4. Suppose the rate of growth of population of fish in Loch Raven reservoir is given by
   \[ \frac{dF}{dt} = \frac{1}{2} F(4 - F) - H \]
   where \( F(t) \) is the fish population at time \( t \) in thousands and \( t \) is time in years and \( H \) is the number of fish harvested (in thousands) per year.
   (a) If no fish are harvested, how many fish can the reservoir support?
   (b) For \( H = 1 \), find and classify the equilibrium solutions. Show your solutions on a phase line.
   (c) If \( H = 1 \) and \( F(0) = 0.5 \)? What happens to the fish population over time?
   (d) What is the bifurcation point for this fish growth model?

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6. Solve
   \[ \frac{dy}{dx} = -\frac{2x^2 + y}{x^2y - x} \]