MA 304: Homework 8 (due Wednesday April 15, 2020)

Reading

Sections 3.7

1. Use variation of parameters to find the general solution to

$$y'' + y' = \frac{1}{1 + e^{-t}}$$

given that $y_1(t) = e^{-t}$ and $y_2(t) = 1$ are solutions to the homogeneous problem.

2. Find a particular solution to

$$y'' + y = \tan(t) + 3t - 1$$

Hint: break up the problem into 2 separate equations and use variation of parameters on one sub-problem and the method of undetermined coefficients on the other

3. A $1kg$ mass attached to a spring of constant $k = 4N/m$ is submerged in water resulting in a large damping constant $\gamma = 5N/m$. Find the position of the mass at time t if

   (a) The mass is lifted $1m$ and released.

   (b) The mass is lifted $1m$ and given a downward velocity of $4m/s$.

4. Given a critically damped spring-mass system described by

$$mu'' + \gamma u' + ku = 0$$
$$u(0) = u_0, \quad u'(0) = 0$$

Show that $\lim_{t\to\infty} u(t) = 0$ but $u(t)$ is never zero.