EXAM 2 REVIEW

You should be able to

1. Solve homogeneous ODEs with constant coefficients

   i.e $ay'' + by' + cy = 0$

* We have coverd 3 cases of this type and solutions for each. Stake them.

* This is Sections 3.1, 3.3 and 3.4. - so a big chunk of Exam 2!

2. Determine the interval of existence of the solution of an initial value problem

$$y'' + p(t)y' + q(t)y = g(t) \qquad (P)$$
$$y(t_0) = y_0, \quad y'(t_0) = y_0'$$

* FACT

If $p(t)$, $q(t)$ and $g(t)$ are continuous on an open interval I that contains the initial condition then the problem has a unique solution on I.

Note

You must write the given ODE in the standard form before applying this fact.

3. Understand the idea of the PRINCIPLE OF SUPERPOSITION

i.e

* If y_1 and y_2 are any 2 solutions of the ODE (P) and

$$W[y_1, y_2](t) = y_1 y_2' - y_1' y_2 \neq 0 \text{ then}$$

every solution can be written as $y = c_1 y_1 + c_2 y_2$.

$\{y_1, y_2\}$ are a fundamental solution set.

2 + 3 + 4 are section 3.2

4. Be able to compute the WRONSKIAN without solving the ODE for y_1 and y_2

* Given $y'' + p(t)y' + q(t)y = 0$, with p, q continuous, then

$$W[y_1, y_2](t) = c e^{-\int p(t)\,dt}$$

; where c is a constant.

(5) Solve non-homogeneous equations using the method of undetermined coefficients

i.e $y'' + p(t)y' + q(t)y = g(t)$

Note

Here we ~~~~ still consider ~~~~ constant coefficients.

* The method relies on choosing a particular form of the solution $y_p(t)$ based on the form of $g(t)$
* The general solution becomes y
* State the cases - The summary of the cases I provided is a good place to start.

(5) - section 3.5.

(6) Solve non-homogeneous equations using the variation of parameters technique

If $g(t)$ does not fit into the categories defined in (5) we can solve the problem

$$y'' + p(t)y' + q(t)y = g(t).$$

* The particular solution takes the form

$$y_p(t) = u_1(t)\, y_1(t) + u_2(t)\, y_2(t)$$

when $\{y_1(t), y_2(t)\}$ form a fundamental solution set.

$$u_1(t) = -\int \frac{y_2(t)g(t)}{W[y_1,y_2](t)}\,dt \quad \text{and} \quad u_2(t) = \int \frac{y_1(t)g(t)}{W[y_1,y_2](t)}\,dt$$

This is section 3.6