Summary - Method undetermined coefficients

In the following, assume that $P_n(t)$ is a polynomial of degree n of the form

$$P_n(t) = A_0 + A_1 t + A_2 t^2 + \cdots A_n t^n$$

1. We can summarize all the cases of the right hand side as follows

   (a) **Polynomials**
   To find a particular solution to

   $$ay'' + by' + cy = g_i(t) = P_n(t)$$

   use the form
   $$y_p(t) = t^s(A_0 + A_1 t + A_2 t^2 \cdots + A_n t^n)$$

   with

   i. $s = 0$ if 0 is not a root of the characteristic polynomial.
   ii. $s = 1$ if 0 is a simple root of the characteristic polynomial
   iii. $s = 2$ if 0 is a double root of the characteristic polynomial

   (b) **Products of polynomials and exponentials**
   To find a particular solution to

   $$ay'' + by' + cy = g_i(t) = P_n(t)e^{rt}$$

   where m is a non-negative integer, use the form

   $$y_p(t) = t^s(A_0 + A_1 t + A_2 t^2 + \cdots + A_n t^n)e^{rt}$$

   with

   i. $s = 0$ if r is not a root of the characteristic polynomial
   ii. $s = 1$ if r is a simple root of the characteristic polynomial
   iii. $s = 2$ if r is a double root of the characteristic polynomial

   (c) **Products of polynomials, exponential and trigonometric functions**
   To find a particular solution to

   $$ay'' + by' + cy = P_n(t)e^{\alpha t}\cos(\mu t) \text{ or } P_n(t)e^{\alpha t}\sin(\mu t)$$

   use the form

   $$y_p(t) = t^s(A_0 + A_1 t + \cdots + A_n t^n)e^{\alpha t}cos(\mu t) + t^s(A_0 + A_1 t + \cdots + A_n t^n)e^{\alpha t}\sin(\mu t)$$

   with

   i. $s = 0$ if $\alpha + i\mu$ is not a root of the characteristic polynomial
   ii. $s = 1$ if $\alpha + i\mu$ is a root of the characteristic polynomial