03/18 Worksheet solutions

1. $y'' - 2y' - 3y = 3e^{2t}$

first solve the homogeneous problem $y'' - 2y' - 3y = 0$

Char poly is $r^2 - 2r - 3 = 0 \iff (r-3)(r+1) = 0 \Rightarrow r = 3,\ r = -1.$

$y_c(t) = c_1 e^{3t} + c_2 e^{-t}$

The rhs is exponential so, let $y_p(t) = Ae^{2t}$ then

$$y_p'(t) = 2Ae^{2t}, \quad y_p''(t) = 4Ae^{2t}$$

and plug that into the ODE

$$4Ae^{2t} - 2(2Ae^{2t}) - 3(Ae^{2t}) = 3e^{2t}$$

$$-3Ae^{2t} = 3e^{2t} \quad \text{so} \quad A = -1$$

This means that the general solution is

$$\boxed{y(t) = c_1 e^{3t} + c_2 e^{-t} - e^{2t}}$$

2. $y'' - y' - 2y = -2t - 4t^2$

The characteristic polynomial for the homogeneous problem is

$$r^2 - r - 2 = 0 \iff (r+1)(r-2) = 0 \quad r = -1, 2$$

$$y_c(t) = c_1 e^{-t} + c_2 e^{2t}$$

The rhs is quadratic so we choose $y_p(t) = At^2 + Bt + C.$

$$y_p'(t) = 2At + B, \quad y_p''(t) = 2A$$

Plug into ODE

$$2A - (2At + B) - 2(At^2 + Bt + C) = -2t - 4t^2$$

$$2A - 2At - B - 2At^2 - 2Bt - 2C = -2t - 4t^2$$

$$(2A - B - 2C) - (2A + 2B)t - 2At^2 = -2A - 4t^2$$

comparing coefficients

t^2

$-2A = -4 \Rightarrow A = 2$

t

$-(2A + 2B) = -2 \Rightarrow -(4+2B) = -2$

$-4 - 2B = -2 \Rightarrow 2B = -2 \Rightarrow B = -1.$

Constants

$2A - B - 2C = 0$

$2(2) - (-1) - 2C = 0$

$4 + 1 = 2C \Rightarrow C = 2/5$

$y_p(t) = 2t^2 - t + \frac{5}{2}$

So the general solution is $\boxed{y(t) = C_1 e^{-t} + C_2 e^{2t} + \left(2t^2 - t + \frac{5}{2}\right)}$

#3

$y'' - 3y' - 4y = 2\sin(t)$

Char polynomial is $r^2 - 3r - 4 = 0 \Leftrightarrow (r+1)(r-4) = 0$, $r = -1, 4$

$y_c(t) = C_1 e^{-t} + C_2 e^{4t}$

To find $y_p(t) = A\sin(t) + B\cos(t)$ see **Example #2 on page 135.**

Notice that the book goes into a small discussion of why we need both $\sin(t)$ and $\cos(t)$ in the general form of $y_p(t)$