Worksheet 03/20 solutions

1. Solve $y'' + 4y = 5t^2 e^t$

Step I

Find $y_c(t)$

$$r^2+4=0 \Rightarrow r = \pm\sqrt{-4} = \pm 2i \Rightarrow \lambda = 0, \mu = 2.$$

$$y_c(t) = c_1 e^{\lambda t}\cos(\mu t) + c_2 e^{\lambda t}\sin(\mu t)$$

$$= c_1 \cos(2t) + c_2 \sin(2t)$$

Step II

Find $y_p(t)$

$$y_p(t) = (At^2 + Bt + C)e^t$$

$$y_p'(t) = (2At + B)e^t + (At^2 + Bt + C)e^t$$

$$y_p''(t) = 2Ae^t + (2At+B)e^t + (2At+B)e^t + te^t(At^2+Bt+C)$$

$$= 2Ae^t + 2(2At+B)e^t + (At^2+Bt+C)e^t$$

$$y_p'' + 4y_p' = \{2Ae^t + 2(2At+B)e^t + (At^2+Bt+C)e^t\} + 4\{(At^2+Bt+C)e^t\}$$

$$= (2A + 2B + \overset{+4C}{C_1})e^t + (4A + B + 4B)e^t + (A + 4A)t^2 e^t$$

$$= (2A + 2B + 5C)e^t + (4A + 5B)te^t + 5At^2e^t = 5t^2e^t$$

Comparing LHS and RHS

$5A = 5 \Rightarrow A = 1$

$2A + 2B + 5C = 0$ ... (i)

$4A + 5B = 0$ ... (ii)

plug in $A = 1$ into (ii) $\Rightarrow 5B = -4 \Rightarrow B = -\frac{4}{5}$

$2A + 2B + 5C = 0 \Rightarrow 2 - \frac{8}{5} + 5C = 0 \Rightarrow C = -\frac{2}{5}$

$y_p(t) = \left(t^2 - \frac{4}{5}t - \frac{2}{25}\right)e^t \Rightarrow y(t) = c_1\cos(2t) + c_2\sin(2t) + y_p(t)$

2. (a) $y(t) = y_1(t) + \frac{1}{3}e^{2t}$

(b) $y(t) = 5\cos(t)$

(c) $y(t) = 4\cos(t) - \frac{18}{3}e^{2t}$