1. Find the form for a particular solution to

\[ y'' + 2y' - 3y = f(t) \text{ and } y'' - 2y' + y = f(t) \]

where \( f(t) \) equals

(a) \( 2te^t \sin t \)
(b) \( t^2 \cos \pi t \)
(c) \( 5e^{-3t} \)
(d) \( t^2e^t \)

*Note: You do not need to actually find the particular solution, just write down the form of the particular solution as I did in Example 7*

**Solutions**

1. The particular solutions to

\[ y'' + 2y' - 3y = f(t) \text{ and } y'' - 2y' + y = f(t) \]

The first thing we need to do is find the complimentary solution. The characteristic polynomial is

\[ r^2 + 2r - 3 = 0 \iff (r - 1)(r + 3) = 0 \implies r = 1, -3 \]

therefore

\[ y_c(t) = c_1e^t + c_2e^{-3t} \]

(a) \( f(t) = 2te^t \sin t \). The rhs is a product of a polynomial, \( 2t \), an exponential, \( e^t \) and a trig function, \( \sin t \) therefore we are in case (c) of the summary list and \( \alpha + i \mu \) (with \( \alpha = 1, \mu = 1 \)) is not a root of the characteristic polynomial, therefore \( s = 0 \) and

\[ y_p(t) = (A_0 + A_1t)e^t \cos(t) + (B_0 + B_1t)e^t \sin(t) \]

(b) \( f(t) = t^2 \cos(\pi t) \). Use the same argument as above

\[ y_p(t) = (A_0 + A_1t + A_2t^2) \cos(\pi t) + (B_0 + B_1t + B_2t^2) \sin(\pi t) \]

(c) \( f(t) = 5e^{-3t} \). Notice that \( r = -3 \) is a simple root of the characteristic polynomial so using case (b) of summary list, \( s = 1 \) and therefore

\[ y_p(t) = Ate^{-3t} \]

(d) \( f(t) = t^2e^t \). Notice that \( r = 1 \) is a simple root of the characteristic polynomial so using case (b) of the summary list, \( s = 2 \) and therefore

\[ y_p(t) = t(A_0 + A_1t + A_2t^2)e^t \]

**Additional Reading/ Examples**

Section 3.5 pages 138–141