Worksheet 03/23 solutions

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1. Find the form for a particular solution to

$$y'' + 2y' - 3y = f(t) \text{ and } y'' - 2y' + y = f(t)$$

   where $f(t)$ equals

   (a) $2te^t \sin t$

   (b) $t^2 \cos \pi t$

   (c) $5e^{-3t}$

   (d) $t^2 e^t$

   Note: **You do not need to actually find the particular solution, just write down the form of the particular solution as I did in Example 7**

Solutions

1. The particular solutions to

$$y'' + 2y' - 3y = f(t) \text{ and } y'' - 2y' + y = f(t)$$

   The first thing we need to do is find the complimentary solution. The characteristic polynomial is

$$r^2 + 2r - 3 = 0 \Leftrightarrow (r-1)(r+3) = 0 \Longrightarrow r = 1, -3$$

   therefore

$$y_c(t) = c_1 e^t + c_2 e^{-3t}$$

   (a) $f(t) = 2te^t \sin(t)$. The rhs is a product of a polynomial, $2t$, an exponential, e^t and a trig function, $sin(t)$ therefore we are in **case (c) of the summary list** and $\alpha + i\mu$ (with $\alpha = 1, mu = 1$) is not a root of the characteristic polynomial, thefore $s = 0$ and

$$y_p(t) = (A_0 + A_1 t)e^t \cos(t) + (B_0 + B_1 t)e^t \sin(t)$$

   (b) $f(t) = t^2 \cos(\pi t)$. Use the same argument as above

$$y_p(t) = (A_0 + A_1 t + A_2 t^2)\cos(\pi t) + (B_0 + B_1 t + B_2 t^2)\sin(\pi t)$$

   (c) $f(t) = 5e^{-3t}$. Notice that $r = -3$ is a simple root of the characteristic polynomial so using **case (b) of summary list**, $s = 1$ and therefore

$$y_p(t) = Ate^{-3t}$$

   (d) $f(t) = t^2 e^t$. Noteice that $r = 1$ is a simple root of the characteristic polynomial so using **case (b) of the summary list**, $s = 2$ and thefore

$$y_p(t) = t(A_0 + A_1 t + A_2 t^2)e^t$$

Additional Reading/ Examples

Section 3.5 pages 138 –141