Worksheet 03/30 Solutions

Problems

1. Find the general solution of

$$t^2y'' - 3ty' + 4y = t^2\ln(t)$$

given that $y_1(t) = t^2$ and $y_2(t) = t^2\ln(t)$ satisfy the homogeneous problem. *First write the ode in standard form by dividing out by* t^2 *to get*

$$y'' - \frac{3}{t}y' + \frac{4}{t^2}y = \ln(t).$$

The functions $y_1(t) = t^2$ *and* $y_2(t) = t^2\ln(t)$ *therefore*

$$W[y_1, y_2] = t^3$$

We can then apply the method of variation of parameters to find a particular solution

$$y_p(t) = u_1(t)y_1(t) + u_2(t)y_2(t)$$

with

$$u_1(t) = \int \frac{t^2\ln(t)\ln(t)}{t^3}\,dt = \int \frac{(\ln(t))^2}{t}\,dt = -\frac{(\ln(t))^3}{3}$$
$$u_2(t) = \int \frac{t^2\ln(t)}{t^3}\,dt = \int \frac{\ln(t)}{t}\,dt = \frac{(\ln(t))^2}{2}$$

Both integrals done via u-sub with $u = \ln(t)$. *We can conclude that the general solution is*

$$y(t) = c_1t^2 + c_2(t^2\ln(t) + (-\frac{(\ln(t))^3}{3})t^2 + (\frac{(\ln(t))^2}{2})(t^2\ln(t)$$

2. A mass of $\frac{1}{2}$ is attached to a spring with spring constant $k = 2\frac{kg}{s^2}$. The spring is pulled down an additional $1m$ and released. Find the equation of motion if the damping constant is $\gamma = 2\frac{kg}{s}$. *The general equation is*

$$mu''(t) + \gamma u'(t) + ku(t) = 0$$

so we can just plug in to get

$$\frac{1}{2}u''(t) + 2u'(t) + 2u(t) = 0$$
$$u(0) = 1, u'(0) = 0$$

Additional Reading/ Examples

Section 3.6 pages 143 -146