Worksheet 04/20 solutions

Problems

1. Use the results from the lecture to find the Laplace transform of

   (a) $7e^{2t}$

   From the lecture notes we have $\mathcal{L}\{e^{at}\} = \dfrac{1}{s-a}$, *therefore* $\mathcal{L}\{7e^{2t}\} = 7\mathcal{L}\{e^{2t}\} = \dfrac{7}{s-2}$

   (b) $2 + 6e^{-4t}$

$$\mathcal{L}\{2 + 6e^{-4t}\} = 2\mathcal{L}\{1\} + 6\mathcal{L}\{e^{-4t}\} = \frac{2}{s} + \frac{6}{s+4}$$

2. Find the Laplace transform of each of the following using the definition

   (a) te^{at}

$$\begin{aligned}
\mathcal{L}\{te^{at}\} &= \int_0^\infty e^{-st}(te^{at})\,dt = \lim_{B\to\infty}\int_0^B te^{a-s}t\,dt \\
&= \lim_{B\to\infty}\left[-\frac{te^{(a-s)t}}{s-a}\bigg|_0^B + \int_0^B \frac{1}{s-a}e^{(s-a)t}\,dt\right] \\
&= \lim_{B\to\infty}\left[\frac{1 - e^{B(a-s)} + B(a-s)e^{B(a-s)}}{(s-a)^2}\right] \\
&= \frac{1}{(s-a)^2} \qquad \textit{provided } s > a
\end{aligned}$$

   (b) $f(t) = \begin{cases} t, & 0 \le t < 1 \\ 1, & 1 \le t < \infty \end{cases}$

$$\begin{aligned}
\mathcal{L}\{f(t)\} &= \int_0^\infty e^{-st}f(t)\,dt = \int_0^1 te^{-st}\,dt + \int_1^\infty e^{-st}\,dt \\
&= -\frac{e^{-s}}{s} - \frac{e^{-s}}{s^2} + \frac{1}{s^2} + \frac{e^{-s}}{s} = -\frac{e^{-s}}{s^2} + \frac{1}{s^2}
\end{aligned}$$

Reading

Section 6.1.