Bisection method

Given an interval $[a, b]$ satisfying $f(a)f(b) < 0$

- *Bisection method*
  - B1. Define $c = (a + b)/2$
  - B2. if $(b - c) \leq \epsilon$, then accept c as the root and stop.
  - B3. if $sign[f(b)] \cdot sign[f(c)] \leq 0$, then set $a = c$. Otherwise, set $b = c$. Return to step B1.

Bisection method

- Find a solution to $f(x) = x^3 + 2x^2 - 3x - 1 = 0$ for $[a, b] = [-3, -2]$ with $\epsilon = 10^{-8}$.

```
>>bisect(-3,-2,1.0e-08)
initial interval (i=0):  a=-3, b=-2, f(a)=-1, f(b)=5
n    a            b              f(c)       ERROR
1   -3.000000,   -2.500000,  3.375000,   2.500000e-01
2   -3.000000,   -2.750000,  1.578125,   1.250000e-01
3   -3.000000,   -2.875000,  0.392578,   6.250000e-02
4   -2.937500,   -2.875000,  -0.277100,  3.125000e-02
                     .
                     .
                     .
23   -2.912229,   -2.912229,  -0.000000, 5.960464e-08
24   -2.912229,   -2.912229,  0.000001,  2.980232e-08
25   -2.912229,   -2.912229,  0.000000,  1.490116e-08
26   -2.912229,   -2.912229,  0.000000,  7.450581e-09
ans = -2.912229165434837
```

- Bisection method converges *linearly* after 26 iterations.
- The error in the solution is approximately $7.450581e-09$.