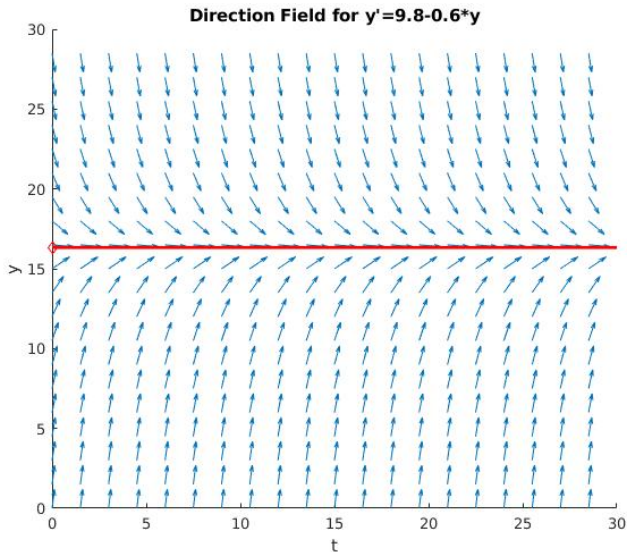


# Direction Fields

## Section 1.1

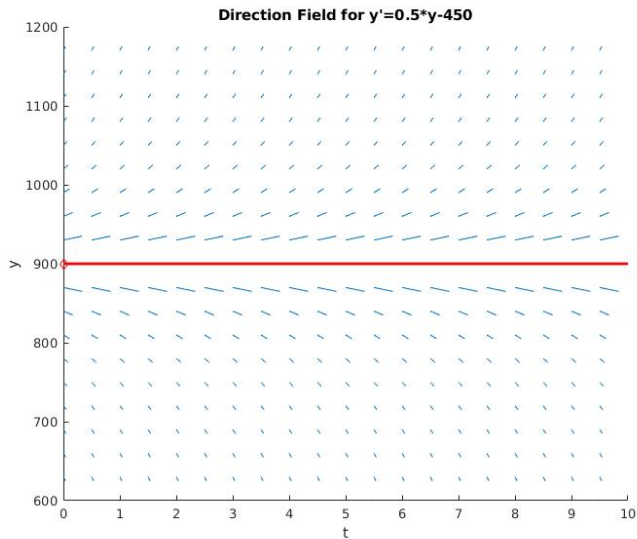
# Falling Sky-diver ( $\frac{dy}{dt} = g - \frac{\gamma y}{m}$ ) : Direction Field



## Falling Sky-diver ( $\frac{dy}{dt} = g - \frac{\gamma y}{m}$ ) : Direction Field

- The arrows represent the slope of the tangent line (acceleration or  $\frac{dy}{dt}$ ) at each point in the  $ty$ -plane.
- All solutions converge to the equilibrium solution,  $(16\frac{1}{3})$  m/s
- $\lim_{t \rightarrow \infty} y(t) = 16\frac{1}{3}$  m/s

# Mice population ( $\frac{dy}{dt} = rp - k$ ) : Direction Field



## Mice population ( $\frac{dy}{dt} = rp - k$ ) : Direction Field

- The arrows represent the slope of the tangent line (rate of growth of population or  $\frac{dy}{dt}$ ) at each point on the  $ty$ -plane.
- All solutions diverge from the equilibrium solution
- If  $y(0) < 900$ ,  $\lim_{t \rightarrow \infty} y(t) = 0$  (Mice all die out!)
- If  $y(0) > 900$ ,  $\lim_{t \rightarrow \infty} y(t) = \infty$  (Mice population explodes!)

# Neurology

- The level of activity of certain nerve cells in the brain can be modelled by an ODE

$$y'(t) = -y(t) + \frac{1}{e^{-15(y(t)-0.5-0.3\cos(2\pi t))}}$$

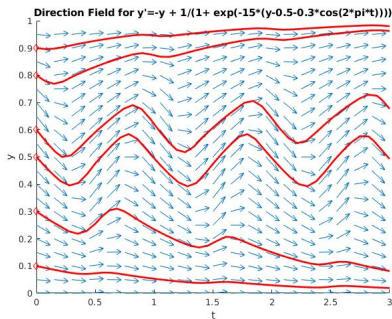
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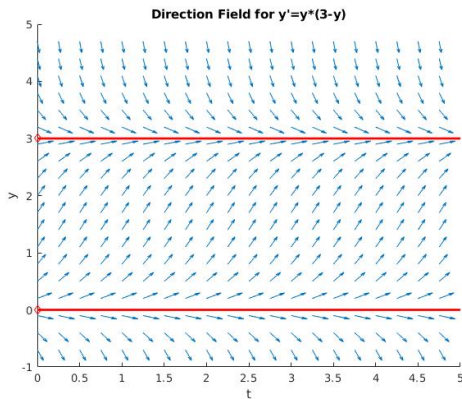
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- The ODE exhibits 3 equilibrium states!

## Other Examples

$$y' = y(3 - y)$$





## Other Examples

$$y' = e^{-t} + y$$

