Direction Fields

Section 1.1
Falling Sky-diver \( \left( \frac{dy}{dt} = g - \frac{\gamma y}{m} \right) \) : Direction Field

Direction Field for \( y' = 9.8 - 0.6y \)
Falling Sky-diver \( (\frac{dy}{dt} = g - \frac{\gamma y}{m}) \): Direction Field

- The arrows represent the slope of the tangent line (acceleration or \( \frac{dy}{dt} \)) at each point in the \( ty \)-plane.
- All solutions converge to the equilibrium solution, \( (16\frac{1}{3}) \) m/s
- \( \lim_{t \to \infty} y(t) = 16 \frac{1}{3} \) m/s
Mice population \( \frac{dy}{dt} = rp - k \) : Direction Field
Mice population \( \left( \frac{dy}{dt} = rp - k \right) \) : Direction Field

- The arrows represent the slope of the tangent line (rate of growth of population or \( \frac{dy}{dt} \)) at each point on the ty-plane.
- All solutions diverge from the equilibrium solution.
- If \( y(0) < 900 \), \( \lim_{t \to \infty} y(t) = 0 \) (Mice all die out!)
- If \( y(0) > 900 \), \( \lim_{t \to \infty} y(t) = \infty \) (Mice population explodes!)
The level of activity of certain nerve cells in the brain can be modelled by an ODE

\[ y'(t) = -y(t) + \frac{1}{e^{-15(y(t)-0.5-0.3\cos(2\pi t))}} \]

where \( y(t) \) is the percentage of cells that are active at time \( t \).
The level of activity of certain nerve cells in the brain can be modelled by an ODE

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The ODE exhibits 3 equilibrium states!
$y' = y(3 - y)$

Direction Field for $y' = y(3 - y)$
Other Examples

\[ y' = e^{-t} + y \]