Final Exam Review
MATH 251.01(02), CALCULUS I, SPRING 2014
In addition to the review from Exams 1 and 2 you are also responsible for the following material:

1. **Applications of differentiation**
   (a) Maxima and minima of functions - Be able to find extreme values of continuous functions on closed intervals.
   (b) Curve sketching – local maxima/minima, second derivative test,determine concavity from second derivative information.
   (c) Indeterminate forms and *L'Hopital's Rule*
   (d) Optimization problems

   Practice Problems

   (a) Section 4.1 – 3,5,7,9,30,43,49,54,55,61
   (b) Section 4.2 – 1,9,17,18
   (c) Section 4.3 – 8,9,12,46
   (d) Section 4.4 – 7,11,13,30,56,57
   (e) Section 4.7 – 16,19,28,58

2. **Integral Calculus**
   (a) Antiderivatives - rules for basic functions
   (b) The area problem - estimate the area under the curve of $y = f(x)$ using *Riemann Sums.*
   (c) Recognize that the area under a curve from a point $x = a$ to $x = b$ is the definite integral $\displaystyle\int_a^b f(x)\,dx$.
   (d) Evaluate integrals – understand the distinction between definite and indefinite integrals.
   (e) Fundamental Theorem of Calculus (FTC) – Suppose f is a continuous function on $[a, b]$ then:
      i. Part I - If we define $g(x) = \int_a^x f(t)\,dt$ then $g'(x) = f(x)$.
      ii. Part II - $\displaystyle\int_a^b f(x)\,dx = F(b) - F(a)$, F is the antiderivative of f.
   (f) Substitution rule.

   Practice Problems

   (a) Section 5.1 – 5
   (b) Section 5.2 – 34,39

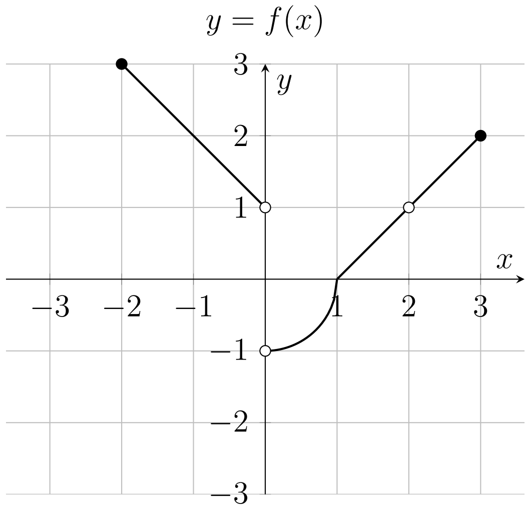
(c) Section 5.3 – 39,40,41,59

(d) Section 5.4 – 11,22,29

(e) Section 5.5 – 3,5,8,9,18,44,45

Practice Exam

1. Consider the graph of $y = f(x)$ shown below



(a) State the *Domain* and *Range* of f.

(b) Use the graph of f to determine the following:

i. $\lim_{x\to 2} f(x)$

ii. $\lim_{x\to 0} f(x)$

iii. $\lim_{x\to 1} 2f(x)$

(c) State with reasons the numbers where f is not *differentiable*.

(d) State with reasons the numbers where f is not *continuous*.

(e) Find $\int_{-1}^{1} f(x)\,dx$

2. State the definition of the *derivative* of a function f.

(a) Use the definition of the derivative to find the derivative of $f(x) = \sqrt{x}$.

3. Use limit laws to calculate the following limits:

(a) $\lim_{x\to 2} \frac{x^2 + x - 6}{x - 2}$

(b) $\lim_{x\to\infty} (e^{-x} + \sin x)$

(c) Suppose $4x - 9 \leq f(x) \leq x^2 - 4x + 7, \quad x \geq 0$. Find $\lim_{x \to 4} f(x)$.
(HINT: Squeeze theorem)

(d) $\lim_{x \to 0} (1 - 4x)^{\frac{1}{x}}$

4. Find $\lim_{x \to 0^+} 2x \cot x$

5. Find the derivative for each of the following functions:

   (a) $y = \dfrac{\sqrt{x}}{1 + e^{-2x}}$

   (b) $y = \sin(e^x) + e^{\sin x}$

   (c) $y = (2x)^x$

   (d) $y = \ln(\cos^2 x)$

6. Find the equation of the tangent line to the curve $\sin(x + y) = 2x - 2y$ at (π, π).

7. Consider the function $F(x) = x^4 - 4x^3$,

   (a) State the domain of F.

   (b) Find and classify the critical points of F.

   (c) On what intervals if F increasing? decreasing?

   (d) Does F have any *inflection* points? On what intervals is F concave up? concave down?

   (e) Sketch a detailed graph of F in the space provided. i.e label the coordinates of critcal points, infection points, and intercepts.

8. Find the point on the line $y = 2x + 3$ that is closest to the origin.

9. Set up the Riemann sum to estimate the area under the graph of $y(x) = 16 - x^2$ between $x = 2$ and $x = 4$ using 4 approximating rectangles and the *left endpoint* rule.

10. You are filling up a cylindrical tank with a radius of $5m$ with water at a rate of $3cm^3/min$. Unbeknownst to you the tank has a hole and is leaking at a rate of $1cm^3/min$! How fast is the height of the water increasing?
The volume of a cylinder $V = \pi r^2 h$.

11. Find the derivative of $g(x) = \displaystyle\int_0^{\ln(x)} e^{\sin x}\, dx$.

12. A bee moves in a straight line and has acceleration given by $a(t) = 6t + 4$. Its initial velocity is $v(0) = -6cm/s$ and its initial displacement $s(0) = 9cm$. Find the position function of the bee $s(t)$.

13. Evaluate the following integrals

(a) $\displaystyle\int_1^4 \frac{\sqrt{x} - x}{x^2}\, dx$

(b) $\displaystyle\int cos^2\theta \sin\theta \, d\theta$

(c) $\displaystyle\int \frac{\sin(\ln x)}{2x}\, dx$