REVIEW

MATH 251, CALCULUS I, SPRING 2014

The following material is covered in prerequisites for this course.

Basic Algebra

1. **Fractions**: Recall fractions are numbers of the form $\frac{a}{b}$ for any real numbers a and b. The following are basic fraction manipulations:

$$\begin{aligned} \frac{a}{b}+\frac{c}{d} &= \frac{ad+bc}{bd} \\ \frac{a}{b}-\frac{c}{d} &= \frac{ad-bc}{bd} \\ \frac{a}{b}\cdot\frac{c}{d} &= \frac{ac}{bd} \\ \frac{a}{b}\div\frac{c}{d} &= \frac{a}{b}\cdot\frac{d}{c} \end{aligned}$$

Example 1 *Simplify the following into a single fraction*

(a) $\frac{1}{2}+\frac{1}{x}$

(b) $x\left(\frac{1+\frac{1}{x}}{x+\frac{1}{x}}\right)$

Solution:

(a) $\frac{1}{2}+\frac{1}{x}=\frac{x+2}{2x}$

(b) $x\left(\frac{1+\frac{1}{x}}{x+\frac{1}{x}}\right)=x\left(\frac{\frac{x+1}{x}}{\frac{x^2+1}{x}}\right)=x\big(\frac{x+1}{x}\cdot\frac{x}{x^2+1}\big)=\frac{x^2+x}{x^2+1}$

2. **Equation of a line:** We recall 2 forms of the equation of a line:

$$\begin{aligned} y &= mx+b \quad \text{slope-intercept form}(m=\text{slope}, b=\text{intercept}) \\ y &= m(x-x_0)+y_0 \quad \text{point-slope form } (m=\text{ slope}, (x_0,y_0)- \text{ a given point on the line }) \end{aligned}$$

3. **Laws of Exponents:** We consider *exponential numbers* (i.e numbers of the form a^b). Recall the following:

$$\begin{aligned} a^{-b} &= \frac{1}{a^b} \\ a^{1/b} &= \sqrt[b]{a} \\ (a^n)^m &= a^{nm} \\ a^n a^m &= a^{n+m} \\ \frac{a^n}{a^m} &= a^{n-m} \\ (ab)^n &= a^n b^n \end{aligned}$$

Note that in general:

$$\begin{aligned} (x+y)^2 &\neq x^2+y^2 \\ \sqrt{x+y} &\neq \sqrt{x}+\sqrt{y} \end{aligned}$$

Example 2 *Use rules of exponents to simplify the following.*

(a) $(-2)^4$

(b) $\frac{x^{17}}{x^{20}}$

(c) $4^{3/2}$

(d) $\sqrt{36x^4}$

Solution

(a) $(-2)^4 = (-2)(-2)(-2)(-2) = 16$

(b) $\frac{x^{17}}{x^{20}} = \frac{1}{x^3}$

(c) $4^{3/2} = (\sqrt{4})^3 = 8$

(d) $\sqrt{36x^4} = 6x^2$

4. **Factoring expressions:** Recall the following:

   (a) To *factor* is to write an expression as a product of 2 or more terms.
   For example we can factor:

   $$x^3 - x \text{ as } x(x^2-1) = x(x+1)(x-1)$$

   (b) *Factoring* is an important technique to use to solve equations. For example, to solve the equation $x^3 - x = 0$ we factor the left hand-side:

   $$\begin{aligned} x^3 - x &= 0 \\ x(x+1)(x-1) &= 0 \end{aligned}$$

   At this point we recall that if $x(x+1)(x-1) = 0$ then either $x = 0$ or $(x+1) = 0$ or $(x-1) = 0$ therefore the equation has 3 solutions $x = 0, -1, 1$.

   (c) Recall that for quadratic equations $ax^2 + bx + c$ we can always use the quadratic formula to solve for the roots:

   $$ax^2 + bx + c = 0 \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Functions

1. **Function Notation:** Recall the most common function notation "$f(x)$". In this notation f is a function that takes in as input a number x in a set called the **Domain** of the function and outputs a number $f(x)$ in the **Range** of the function.

   Example 3 *Given* $f(x) = x^2 + 1$
   Find:

   (a) *Find* $f(2)$
   (b) *Find* $f(x+h)$
   (c) *Find* $f(3x)$

   Solution:

   (a) $f(2) = 2^2 + 1 = 5$
   (b) $f(x+h) = (x+h)^2 + 1 = x^2 + 2hx + h^2 + 1$
   (c) $f(3x) = (3x)^2 + 1 = 9x^2 + 1$

2. **Function Combinations:** Recall that we can combine functions to obtain new functions.

   Example 4 *Let* $f(x) = 3x^2 + 1$ *and* $g(x) = \cos(x)$.

   (a) *Find a formula for* $f(x) + g(x)$.
   (b) *Find* $f(\pi) - g(\pi)$.
   (c) *Find a formula for* $f(x)/(g(x))^2$.
   (d) *Find* $f(g(x))$.

   Solution:

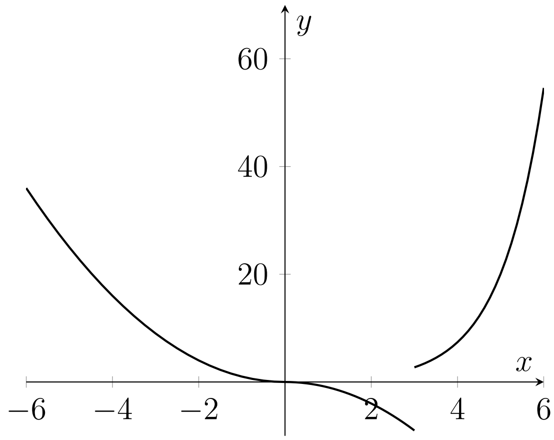
   (a) $f(x) + g(x) = 3x^2 + 1 + \cos(x)$.
   (b) $f(\pi) - g(\pi) = 3(\pi)^2 + 1 + \cos(\pi) = 3\pi^2 + 1 - 1 = 3\pi^2$
   (c) $f(x)/(g(x))^2 = \dfrac{3x^2+1}{(\cos(x))^2} = \dfrac{3x^2+1}{\cos^2(x)}$.
   (d) $f(g(x)) = 3(\cos(x))^2 + 1 = 3\cos^2(x) + 1$

3. **Piecewise Functions** A **piecewise** function is one for which parts of the domain are defined by different functions.

   Example 5 *Let* $f(x)$ *be defined by the following*

$$f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ -x^2 & \text{if } 0 < x \leq 3 \\ e^{x-2} & \text{if } 3 < x \end{cases}$$

$y = f(x)$



4. **Special Functions**

   (a) **Polynomials:**

   - Sums of powers of x (using only whole numbers) along with *coefficients*.
   - Examples – $5x^2$, $-3x + 5$, $-3x^2 + 2x + 1$.
   - In general we can write polynomials are of the form $a_n x^n + \cdots a_1 x + a_0$ where a_i are the coefficients.

   (b) **Exponential and Logarithmic Functions:**

   - Functions of the form a^x for any constant a, (e.g $2^x, 10^x, (1/2)^x, e^x$).
   - Logarithmic functions: functions of the form $(\log_2(x), \log_{10}(x), \ln(x))$.

   (c) **Trigonometric Functions** We will measure an angle θ in radians. The radian measure of an angle θ is

$$\theta \text{ rad } = \frac{s}{r}$$

were s is the arc length and r is the radius of the sector with angle θ.

Example 6 *i. Use a circle of radius* 1 *to calculate the radian measure of* 360 *degrees.*

ii. Find the radian equivalents of the following angles.

$$0, 30, 45, 60, 90, 180, 270$$

Solution

i. We have a circle of radius $r = 1$ therefore $s = 2\pi$. The radian measure of 360

$$\theta = \frac{2\pi}{1} = 2\pi \text{ rad }.$$

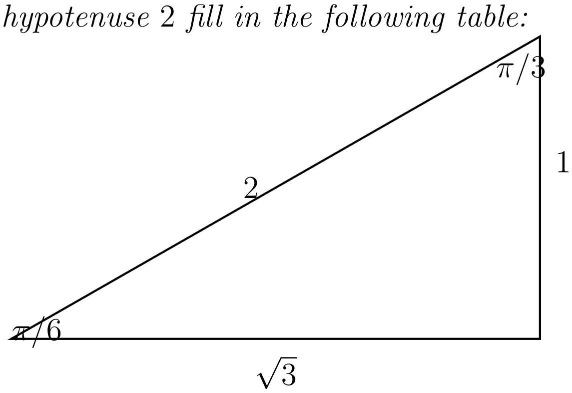
ii. *Converting between degrees and radians*

$$360^\circ = 2\pi \text{ rad} \qquad \times \frac{1}{2} \Longrightarrow 180^\circ = \pi \text{ rad}$$
$$180^\circ = \pi \text{ rad} \qquad \times \frac{1}{2} \Longrightarrow 90^\circ = \frac{\pi}{2} \text{ rad}$$
$$180^\circ = \pi \text{ rad} \qquad \times \frac{1}{6} \Longrightarrow 30^\circ = \frac{\pi}{6} \text{ rad}$$
$$90^\circ = \frac{\pi}{2} \text{ rad} \qquad \times \frac{1}{2} \Longrightarrow 45^\circ = \frac{\pi}{4} \text{ rad}$$
$$90^\circ = \frac{\pi}{2} \text{ rad} \qquad \times 3 \Longrightarrow 270^\circ = \frac{3\pi}{2} \text{ rad}$$

iii.

Values of basic trigonometric functions

Example 7 *Using a right angled triangle with sides of length* 1, $\sqrt{3}$ *and hypotenuse* 2 *fill in the following table:*

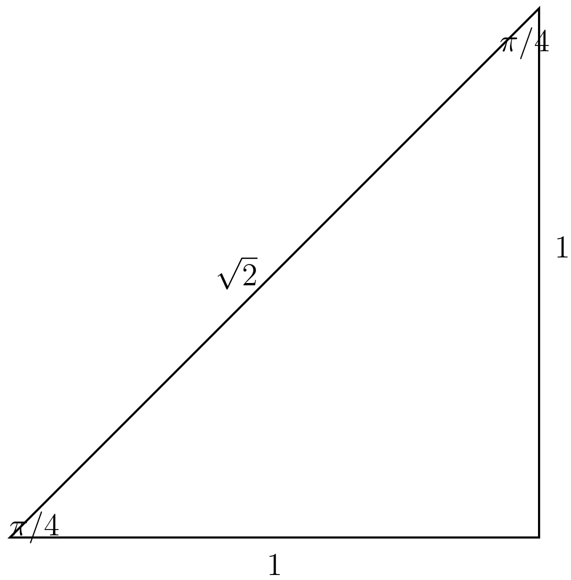


$\sin\left(\frac{\pi}{6}\right)$	=	$\sin\left(\frac{\pi}{3}\right)$	=
$\cos\left(\frac{\pi}{6}\right)$	=	$\sin\left(\frac{\pi}{3}\right)$	=
$\tan\left(\frac{\pi}{6}\right)$	=	$\tan\left(\frac{\pi}{3}\right)$	=

Solution:

$\sin\left(\frac{\pi}{6}\right)$	$= 1/2$	$\sin\left(\frac{\pi}{3}\right)$	$= \sqrt{3}/2$
$\cos\left(\frac{\pi}{6}\right)$	$= \sqrt{3}/2$	$\sin\left(\frac{\pi}{3}\right)$	$= 1/2$
$\tan\left(\frac{\pi}{6}\right)$	$= 1/\sqrt{3}$	$\tan\left(\frac{\pi}{3}\right)$	$= \sqrt{3}$

Example 8 *Using a right angled triangle with sides of length* 1, 1 *and hypotenuse* $\sqrt{2}$ *fill in the following table:*



$\sin\left(\frac{\pi}{4}\right)$	$=$
$\cos\left(\frac{\pi}{4}\right)$	$=$
$\tan\left(\frac{\pi}{4}\right)$	$=$

Solution:

$\sin\left(\frac{\pi}{4}\right)$	$= 1/\sqrt{2}$
$\cos\left(\frac{\pi}{4}\right)$	$= 1/\sqrt{2}$
$\tan\left(\frac{\pi}{4}\right)$	$= 1$