Exam 1 Review
MATH 251.01(02), CALCULUS I, SPRING 2014

- **Functions**

  1. The definition of a function
  2. Determining the domain and range of a function from a graph or the formula of the given function.
  3. When is a curve in the xy plane the graph of a function? - It must pass the vertical line test.
  4. Graphs of common functions covered in class including piecewise functions
  5. Inverse of a function - when is a function f invertible? (It must pass the horizontal line test!)

  Practice Problems:

  1. Section 1.1: 7-10,31-37,49
  2. Section 1.3: 2, 29

- **Limits**

  1. The tangent problem - recall that the slope of a tangent line of $y = f(x)$ at a point P is the limit of the slopes of secant lines passing through P and another point Q that lies on the graph.
  2. Find $\lim_{x \to a} f(x)$ from a graph of the function.
  3. Find limits using limit calculation laws.
  4. Limits at infinity and horizontal asymptotes
  5. In general if a limit does not exist you should be able to clearly justify why it does not exist. For example by comparing the left and right-hand limits at a given point or in some cases the function osscilates (recall the example of $\lim_{x \to 0} \sin(1/x)$ – we said the limit does not exist because the function osscilates between -1 and 1 as $x \to 0$

  Practice Problems:

  1. Find the equation of the tangent line to $y = \dfrac{1}{1-x}$ at the point $P = (2, -1)$ (Use limit laws for calculating your limit)
  2. Given that the displacement of a particle $s(t) = \sqrt{t}$. Find the instantaneous velocity of the particle at $t = 1$. (Again, use limit laws to compute any limits)
  3. Section 2.2: 4,6,8,12,15,31

4. Section 2.3: 1,14,19,22,27,32,37

- **Continuity**

1. Definitions: continuity, left and right continuity at a point.
2. Be able to classify the different types of discontinuities
3. Be able to justify why a function is discontinuous at a given point - The easiest strategy for this is to appeal to the definition (A function is continuous at $x = a$ if $\lim_{x \to a} f(x) = f(a)$.
4. Be able to prescribe ways of fixing removable discontinuities and other discontinuous piecewise functions (see problem 45 in Section 2.5)
5. Intermediate value theorem - Given a function $y = f(x)$ can you determine whether the function f has a root on a given interval.

Practice Problems

1. Section 2.5: 3,4,11,18,23,45,51,5

- **Derivatives**

1. The definition of the derivative of f at a point a denoted $f'(a)$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

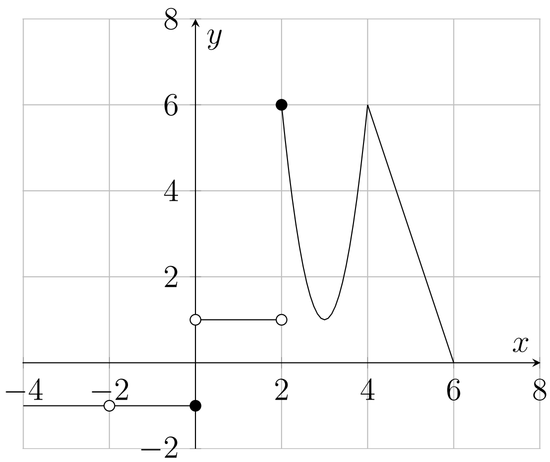
2. Calculate the derivative of a function from the definition
3. Use the derivative to find the instantaneous velocity given a displacement function.

Practice Problems

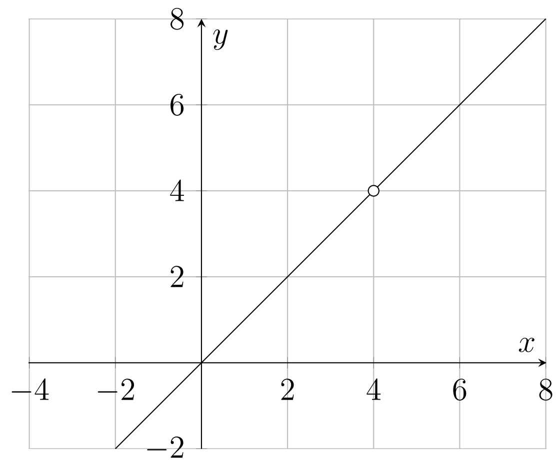
1. Section 2.7: 5,17,23

Sample Exam

$y = f(x)$



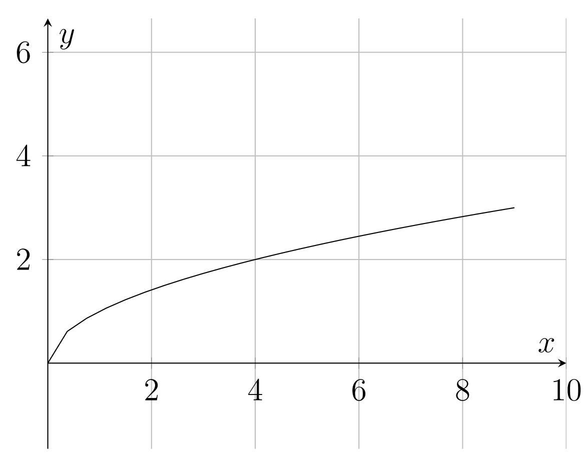
$y = g(x)$



1. (a) State the domain and range of the function f.
   (b) Use the graphs of f and g to evaluate $\lim_{x \to 4}[f(x) + g(x)]$.
   (c) Use the graphs of f to evaluate $\lim_{x \to 2}[3f(x)]$.
   (d) State the numbers at which f discontinuous. For each number state whether f is continuous from the right, or from the left or neither.
   (e) Is the function f invertible? Explain.

2. The graph of the function $y = \sqrt{x}$ is shown below:

$y = \sqrt{x}$



   (a) Draw the tangent line to $f(x) = \sqrt{x}$ at $x = 1$.
   (b) Find the equation of the tangent line to $y = \sqrt{x}$ at $x = 1$.

3. Evaluate the following limits. If a limit does not exist justify your reasoning.

   (a) $\lim_{x\to 3} \frac{x^2 - 2x - 3}{x - 3}$

   (b) $\lim_{x\to -1} \frac{x - 3}{x^2 - 2x - 3}$

   (c) $\lim_{x\to 0^+} \sqrt{x}\cos(\frac{1}{x})$

   (d) $\lim_{x\to \infty} (e^{-x} + 2\sin(\pi x))$

   (e) $\lim_{x\to \infty} \frac{\sqrt{x} + 5}{3\sqrt{x} - 2}$

4. Show that $4x^3 + 6x^2 + 3x - 2 = 0$ has a root on the interval $(0, 1)$.

5. Let $H(x) = \frac{x^2 - x - 2}{x - 2}$ and $G(x) = x + 1$.

   (a) Is $G(x) = H(x)$?

   (b) Is the function $H(x)$ continuous on $(-\infty, \infty)$? If not how can $H(x)$ be made continuous?

6. Let

$$f(x) = \begin{cases} c\cos(x) + e & \text{if } x < \pi \\ x & \text{if } x \geq \pi \end{cases}$$

   Find the value of c that makes the function f continuous on $(-\infty, \infty)$.

7. A projectile moves in a straight line. Its displacement at time t is given by the function $s(t) = \frac{1}{1 + t}$.

   (a) Find the average velocity on the interval $[1, 2]$.

   (b) State the definition of the derivative of a function s at a point $t = a$.

   (c) Find the instantaneous velocity of the projectile at $t = 1$.