Exam 2 Review
MATH 251.01(02), CALCULUS I, SPRING 2014

1. **The derivative**

   (a) Definition of derivative

   $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

   (b) *What does it mean for a function f to be differentiable at a point $x = c$?*

   $$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h} \textit{ has to exist}$$

   (c) Recall ways a function can fail to be differentiable at a point.

   (d) The derivative of f at $x = c$ is the slope of the tangent line to $y = f(x)$ at the point $(c, f(c))$.

   (e) Understand the relationship between the graph of a function $f(x)$ and its derivative $f'(x)$

   Practice Problems

   (a) Section 2.8: 3,9,11,26,37

2. **Rules of differentiation**

   (a) *Power Rule:* $\frac{d}{dx}(x^n) = nx^{n-1}$ for any real number n.

   (b) *Product Rule:* $\frac{d}{dx}(fg) = f'g + g'f$.

   (c) *Quotient Rule:* $\frac{d}{dx}(\frac{f}{g}) = \frac{gf' - g'f}{g^2}$.

   (d) Derivatives of basic trigonometric functions

   (e) Limits involving trigonometric functions

      i. $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

      ii. $\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0$

   (f) *Chain Rule:* $\frac{d}{dx}(f(g(x)) = f'(g(x)) \cdot g'(x)$.

   (g) Implicit differentiation

   (h) Derivatives of Logarithmic functions.

   (i) Logarithmic differentiation

   Practice Problems

(a) Section 3.1: 16,23,27,37,67

(b) Section 3.2: 3,19,24,27,33,47

(c) Section 3.3: 5,8,17,19,24,32,39,40,45

(d) Section 3.4: 7,9,16,21,25,34,35,51,55a

(e) Section 3.5: 2,13,20,32,39,52

(f) Section 3.6: 2,4,19,23,32,34,43,44,48

3. **Applications of derivatives:**

   (a) Understand the derivative as a rate of change

   (b) velocity function as a derivative of the displacement function

   (c) Acceleration function as a derivative of the velocity function.

   Practice Problems

   (a) Section 3.7: 7,8,31

4. Related Rates
   Practice Problems

   (a) Section 3.9: 20,27,38

5. Linear approximations.
   Practice Problems

   (a) Section 3.10: 1,2,24,25

6. Maxima and minima of functions.
   Practice Problems

   (a) Section 4.1: 3-6,30,37,28,41,44,49,55,56,61

Sample Exam 2

1. State the definition of the derivative of a function $y = f(x)$.
   The graph of a function $y = f(x)$ is shown below:

2. Sketch the graph of the derivative $f'(x)$ on the same figure above.

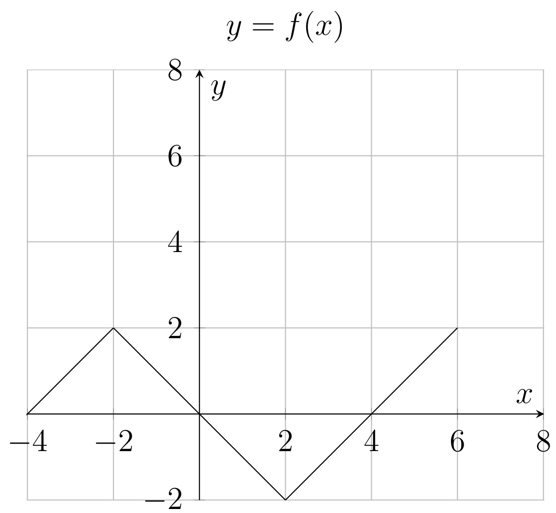
3. Sketch the graph of a *continuous* function f with the following properties:

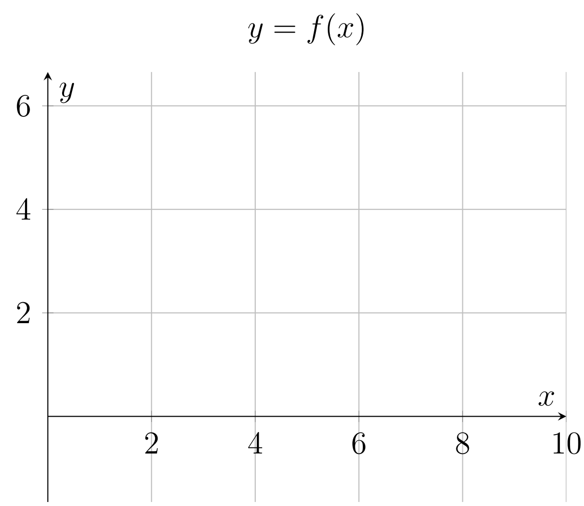
   (a) f is not *differentiable* at $x = 0$.

   (b) f has a horizontal tangent at $x = 2$.

   (c) f has no *absolute maximum* value.

   (d) f has an *absolute minimum* value.





4. Find the derivative for each of the following functions:

   (a) $y = \dfrac{e^{\sqrt{x}} + e^5}{x}$

   (b) $y = \sin\left(\sqrt{x^3 + 2x}\right)$

   (c) $y = x^x$

   (d) $y = \ln\cos^2(x)$

   (e) $y = \tan^{-1}(\sqrt{x})$

5. Find the equation of the *normal* line to the curve $x^3 + y^3 = 6xy$ at the point $(3, 3)$

6. Prove that $\dfrac{d}{dx}(\cot x) = -\csc^2(x)$

7. Find $\lim\limits_{x\to 0} x \cot x$.

8. If you drop a pebble in a pond. It makes waves that dampen with distance. At a particular point x the height of the wave is given by the function

$$h(x) = \frac{\sin(x)}{x}$$

   (a) Find the Linearization of $h(x)$ at the point $a = \frac{\pi}{2}$ inches.

   (b) Use the Linearization to estimate $h(\pi)$ the height a wave makes π inches from the point of impact.

9. You are sitting on a dock, 6 feet above the water and using a rope to pull in a floating buoy. You pull the rope at a speed of $2ft/s$

   (a) Sketch a picture of the situation and set up an equation that relates the distances involved.

   (b) How fast is the horizontal distance between the buoy and the dock changing when the rope is 12 feet long?

10. Find the absolute minimum and absolute maximum value of $f(x) = \ln(x^2 + x + 1)$ on the interval $[-1, 1]$