Solutions to Suggested problems Sections: 3.1 - 3.6
MATH 251.03(04), CALCULUS I, FALL 2013

- Section 3.1

4. $\frac{d}{dx}e^5 = 0$. e^5 is a constant.
8. $f'(t) = 7t^4 - 5t$.
19. $y' = 3(e^x) + 4(-\frac{1}{3}x^(- 4/3)$
23. $y' = x^{3/2} + 4x^{1/2} + 3x^{-1/2}$
25. $f'(x) = 2.4x^{1.4} + 0$
30. $v' = 1 + 2(\frac{1}{6}x^{-5/6} - \frac{2}{3}x^{-5/6}$
35. $y' = 4x^3 + 2e^x$ Equation of normal is $y = -\frac{1}{2}x + 2$.
36. $y' = 2x - 4x^3$. Equation of the normal $y = \frac{1}{2}x - \frac{1}{2}$.
48. $v(t) = s'(t) = 4t^3 - 6t^2 + 2t - 1$, $a(t) = v'(t) = 12t^2 - 12t + 2$, $a(1) = 2m/s^2$.
67. f is not differentiable at 1.
68. g is differentiable at 0 but not at $x = 2$.

- Section 3.2

2. $F'(x) = 2x - 5 - \frac{3}{2}x^{-5/2}$
19. $y' = 2v - v^{-1/2}$
27. $f'(x) = x^4e^x + e^x4x^3$
28. $f'(x) = x^{5/2}e^x + e^x \cdot \frac{5}{2}x^{3/2}$.
29. $f'(x) = \frac{(1+2x)(2x) - x^2(2)}{(1+2x)^2}$
30. $f'(x) = \frac{(x^2-1)(1) - x(2x)}{(x^2-1)^2}$
34. $f'(x) = \frac{(x^2+1)(2) - 2x(2x)}{(x^2+1)^2}$ At $(1,1)$ $f'(1) = 0$ so the equation of the tangent line is $y - 1 = 0(x - 1)$.
43. a. $(fg)'(5) = f(5)g'(5) + g(5)f'(5) = -16$
    b. $(\frac{f}{g})' = \frac{g(5)f'(5) - f(5)g'(5)}{[g(5)]^2} = \frac{-20}{9}$
    c. $(\frac{g}{f})' = \frac{f(5)g'(5) - g(5)f'(5)}{[f(5)]^2} = 20.$
45. $f'(x) = e^xg'(x) + g(x)e^x \Rightarrow f'(0) = 7$.
48. $f'(x) = x^2f(x) \Rightarrow f''(x) = x^2f'(x) + f(x) \cdot 2x$. $f'(2) = 4(10) \Rightarrow f''(2) = 4(40) + 10(4) = 200$

52. a. $y' = x^2 f'(x) + f(x)(2x)$

    b. $y' = \dfrac{x^2 f'(x) = f(x)(2x)}{(x^2)^2}$

    c. $y' = \dfrac{f(x)(2x) - x^2 f'(x)}{[f(x)]^2}$

    d. $y' = \dfrac{\sqrt{x}[xf'(x) + f(x)] - [1 + xf(x)]\dfrac{1}{2\sqrt{x}}(\sqrt{x})^2}{(\sqrt{x})^2}$

53. $f'(x) = \dfrac{1}{(x+1)^2}$ The equation of the tangent line at a point $x = a$ is $y - \dfrac{a}{a+1} = \dfrac{1}{(a+1)^2}(x - a)$. This tangent line passes through $(1, 2)$ when $2 - \dfrac{a}{a+1} = \dfrac{1}{(a+1)^2}(1 - a)$ Solve this equation for $a = -2 \pm \sqrt{3}$ so there are 2 such tangent lines since we have to values of a.

- Section 3.3

5. $y' = \sec\theta(\sec^2\theta + \tan\theta)(\sec\theta\tan\theta)$
8. $f'(t) = \dfrac{\csc^2 t + \cot t}{e^t}$
9. $y' = \dfrac{(2 - \tan x)(1) - x(-\sec^2 x)}{(2 - \tan x)^2}$
14. $y' = \dfrac{\sec x(1 - \sec x)}{\tan^2 x}$
16. $f'(x) = x\sin x(2\tan x + x + s\sec^2 x)$
17. $\dfrac{d}{dx}(\csc x) = -\csc x\cot x$
19. $\dfrac{d}{dx}(\cot x) = -\csc^2 x$
24. $y'(x) = 1 + \sec^2 x \Rightarrow y'(\pi) = 1 + (-1)^2 = 2$. The equation of the tangent line to the given curve at (π, π) is $y - \pi = 2(x - \pi)$
32. a. $2 - \sqrt{3}$

    b. $\dfrac{1 - 2\sqrt{3}}{16}$
33. $x = (2n+1)\pi \pm \dfrac{\pi}{3}$, n is an integer.
34. $x = \dfrac{\pi}{4} + n\pi$
39. 3
40. $\dfrac{2}{3}$
41. 3
45. $\dfrac{1}{2}$
46. 0

52. a. 1
    b. 0

- Section 3.4

7. $F'(x) = 5(x^4 + 3x^2 - 2)^4(4x^3 + 6x)$

16. $y'(x) = e^{-2t}(-\sin 4t \cdot 4) + \cos 4t[e^{-2t}(-2)]$

21. $y'(x) = 3\big(\dfrac{x^2+1}{x^2-1}\big)^2 \cdot \dfrac{(x^2-1)(2x) - (x^2+1)(2x)}{(x^2-1)^2}$

29. $F'(t) = e^{t\sin 2t}(t \cdot 2\cos 2t + \sin 2t \cdot 1)$

34. $y' = x^2 e^{-1/x}(\dfrac{1}{x^2}) + e^{-1/x}(2x)$

35. $y' = -\sin\big(\dfrac{1-e^{2x}}{1+e^{2x}}\big) \cdot \dfrac{(1+e^{2x}(-2e^{2x} - (1-e^{2x})(2e^{2x})}{(1+e^{2x})^2}$

51. $y' = 20(1+2x)^9$

53. $y = -x + \pi$

55a. $y = \dfrac{1}{2}x + 1$

59. $(\dfrac{\pi}{2} + 2n\pi, 3)$ and $(\dfrac{3\pi}{2} + 2n\pi, -1)$, n is an integer.

79. $s'(t) = \dfrac{5\pi}{2}\cos(10\pi t) cm/s.$

- Section 3.5

2. $y' = \dfrac{4x + y + 1}{x}$

13. $y' = \tan x \tan y$

20. $y' = \dfrac{(1+x^2)\sec^2(x-y) + 2x\tan(x-y)}{1 + (1+x^2)\sec^2(x-y)}$

21. $f'(1) = -\dfrac{16}{13}$

25. Equation of tangent line is $y - \dfrac{\pi}{4} = \dfrac{1}{2}(x - \dfrac{\pi}{4})$.

32. Equation of tangent line is $y = -2$

39. $y'' = \dfrac{1}{e^2}$.

49. $y' = \dfrac{2\tan^{-1} x}{1+x^2}$

60. $y' = \dfrac{-1}{2\sqrt{1-x^2}}$

- Section 3.6

2. $f'(x) = \ln x$.

4. $f'(x) = 2\cot x$.

5. $f'(x) = -\dfrac{1}{x}$

19. $y' = -\dfrac{x}{1+x}$

21. $y' = \dfrac{1}{\ln 10} + \log_{10} x$

22. $y' = -\log_2(e) - \dfrac{\pi}{\ln 2}\tan \pi x$

23. $y'' = 3 + 2\ln(2x)$

32. 1

36. 0

37. $c = 7$

38. $a = e^2$

42. $y' = \sqrt{x}e^{x^2-x}(x+1)^{2/3}\big(\dfrac{1}{2x} + 2x - 1 + \dfrac{2}{3x+3}\big)$

43. $y' = x^x(1 + \ln x$

47. $y' = (\cos x)^x(\ln \cos x - x \tan x)$