Theorem 14 (L'Hospital's Rule). If $\lim \dfrac{f(a)}{g(a)}$ equals $\dfrac{0}{0}$ or $\dfrac{\pm\infty}{\pm\infty}$ (and $\lim \dfrac{f'(x)}{g'(x)}$ equals a finite number or $\pm\infty$) then

$$\lim \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)}.$$

Comment. (1) In the previous statement, and the ones that follow, "lim" means any of the following

$$\lim \text{ means } \begin{array}{c} \lim\limits_{x\to a}, \lim\limits_{x\to a^+}, \lim\limits_{x\to a^-} \\ \lim\limits_{x\to\infty}, \lim\limits_{x\to-\infty} \end{array}$$

(2) Note that you do *not* take the quotient rule, and you do *not* take the derivative of $\dfrac{1}{g(x)}$, but rather the derivative of $g(x)$ all by itself. For example, applying L'Hospital's rule to $\dfrac{\ln(x)}{x}$ gives $\dfrac{1/x}{1}$.

Corollary (Variations). There are three common variations of L'Hospital's Rule.

(1) Finding $\lim f(x) \cdot g(x)$.

If $\lim f(x) \cdot \lim g(x)$ equals $0 \cdot \infty$ or $\infty \cdot 0$ then rewrite $f(x)g(x)$ as one of the following fractions

$$f(x)g(x) = \frac{f(x)}{1/g(x)} \quad \text{or} \quad \frac{g(x)}{1/f(x)}$$

and apply L'Hospital's rule to the fraction.

(2) Finding $\lim(f(x) - g(x))$.

If $\lim f(x) - \lim g(x)$ equals $\infty - \infty$ then rewrite $f(x) - g(x)$ as a single fraction and apply L'Hospital's rule.

(It's not always obvious how to turn $f(x) - g(x)$ into a fraction. If $f(x)$ or $g(x)$ is a fraction (including $\sec(x)$, $\tan(x)$, etc.), then combine both terms with a common denominator. If $f(x)$ or $g(x)$ has a square root, write $\dfrac{f(x) - g(x)}{1}$ and then conjugate this fraction.)

(3) Finding $\lim f(x)^{g(x)}$.

If $(\lim f(x))^{(\lim g(x))}$ equals 0^0 or 1^∞ or ∞^0, then do:

(a) Set $y = f(x)^{g(x)}$
(b) Take $\ln$ to get $\ln(y) = g(x)\ln(f(x))$
(c) Find $\lim g(x) \cdot \ln(f(x))$.
(d) Give the final answer

$$\lim y = \lim e^{\ln(y)} = e^{\lim \ln(y)} = e^{\lim g(x)\ln(f(x))}.$$
