Derivatives

Here all the derivative rules and techniques that we have learned this semester. With them, you can take the derivative of any function that you have ever seen! (Note: there are other functions in the world, that you don't know how to take the derivative of, but these involve definitions and formulas that you have probably never seen.)

Formulas marked with a "*" should definitely be memorized.

Basic functions

$$* \frac{d}{dx}x^n = nx^{n-1} \text{ for all real numbers } n$$

The derivatives of x^2, x^3, $\sqrt{x} = x^{1/2}$ were found in 2.8, using the definition of derivative. The derivatives of $1/x = x^{-1}$, $1/x^2 = x^{-2}$ were found in 3.1, again using the definition. The derivative of x^n, for all real numbers n, was found in 3.6 by using logarithmic differentiation.

$$* \frac{d}{dx}\sin(x) = \cos(x) \qquad \frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$
$$* \frac{d}{dx}\cos(x) = -\sin(x) \qquad \frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$
$$* \frac{d}{dx}\tan(x) = \sec^2(x) \qquad \frac{d}{dx}\cot(x) = -\csc^2(x)$$

The derivatives of $\sin(x)$ and $\cos(x)$ were found in 3.3, using the definition, trig identities, and the special limits $\lim_{h\to 0}\frac{\sin(h)}{h} = 1$ and $\lim_{h\to 0}\frac{\cos(h)-1}{h} = 0$. The rest of these derivatives were found in 3.3 using the quotient rule.

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\sec^{-1}(x) = \frac{1}{x\sqrt{x^2-1}}$$
$$\frac{d}{dx}\cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\csc^{-1}(x) = \frac{-1}{x\sqrt{x^2-1}}$$
$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}\cot^{-1}(x) = \frac{-1}{1+x^2}$$

All of these derivatives were found in 3.5, by using implicit derivatives to obtain a relationship between each of these functions and the noninverse function.

$$* \frac{d}{dx}e^x = e^x \text{ and } * \frac{d}{dx}\ln|x| = \frac{1}{x}$$

The derivative of e^x was found in 3.1 using the special limit $\lim_{h\to 0}\frac{e^h-1}{h} = 1$. The derivative of $\ln(x)$ was found in 3.6 using implicit derivatives.

Combinations

$$*(f+g)' = f' + g' \qquad *(f\cdot g)' = f'\cdot g + f\cdot g'$$
$$*(f-g)' = f' - g' \qquad *(f/g)' = \frac{f'\cdot g - f\cdot g'}{(g)^2}$$

$$*(f(g(x)))' = f'(g(x))\cdot g'(x)$$

$$\Big(\text{the combination } f(x)^{g(x)} \text{ is described below}\Big)$$

Here is how some of our basic functions look when combined with the chain rule:

$$* \frac{d}{dx}\Box^n = n\Box^{n-1}\cdot\Box' \qquad * \frac{d}{dx}\sin(\Box) = \cos(\Box)\cdot\Box'$$
$$* \frac{d}{dx}e^{\Box} = e^{\Box}\cdot\Box' \qquad * \frac{d}{dx}\cos(\Box) = \sin(\Box)\cdot\Box'$$
$$* \frac{d}{dx}\ln(\Box) = \frac{1}{\Box}\cdot\Box' \qquad * \frac{d}{dx}\tan(\Box) = \sec^2(\Box)\cdot\Box'$$

You should imagine putting other functions inside the boxes. Of course, the exact same stuff should go inside each box in one of these equations.

The derivatives of $f \pm g$ were proven in 3.1 using basic definitions. The product and quotient rules were proven in 3.2 using the basic definition and some tricks. The chain rule was proven in 3.4, also using the defintiion and some tricks.

Techniques

Implicit derivatives. (1) You start with an equation involving x's, y's, and/or numbers. (2) You take the derivative of both sides of the equation. When you do this you view y as a function of x and use the chain rule, product rule, etc. as needed. (For example $\frac{d}{dx}y^2 = 2yy'$, $\frac{d}{dx}\sin(y) = \cos(y)y'$, $\frac{d}{dx}xy = y + xy'$, etc..) (3) You solve the new equation for y'.

Logarithmic differentiation. (1) You start with a function, often of the form $y = f(x)^{g(x)}$. (2) Take ln of both sides, bring the exponent down in front. (3) Take the implicit derivative. (4) Solve for y'.