251 Midterm 1 preview: Fall, 2007

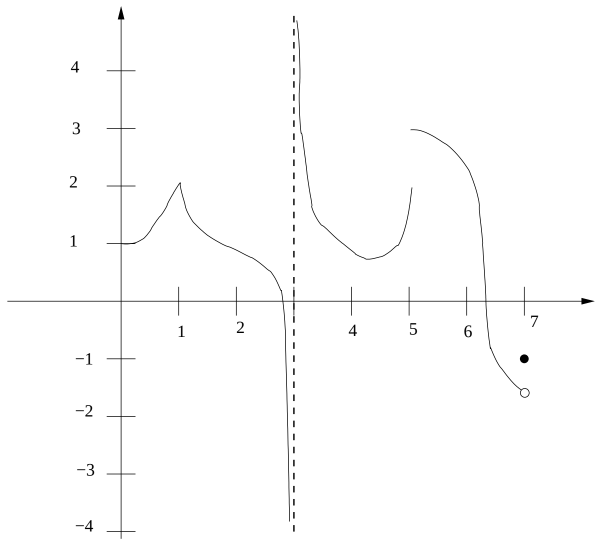
Things to keep in mind as you take this practice test:

- The real test will not be this long. It will probably have around 8 problems.
- You should know/memorize/write down rules for exponents, values of sin, cos and tan at 0, $\pi/6$, $\pi/4$, $\pi/3$ and $\pi/2$ (as well as how to use these angles to move around the unit circle), basic methods of algebra (like foiling, simplifying etc), how to use e^x and $\ln(x)$, how to solve various equations for x.
- You should know/memorize the short cut derivitave rules for all powers of x (i.e. $\frac{d}{dx}x^n$), e^x, the sum rule, constant rule, the product rule, and the quotient rule. You can use the rules at any time unless the problem says otherwise.
- You should know that the derivative is used to find slopes of tangent lines and/or instantaneous velocity.
- You should know how to find limits using tables of numbers, graphs, and/or algebra.
- Since this test is for practice you should think about doing variations of some of the problems, especially the ones that you find difficult.
- Everything should be done algebraicly unless explicitly stated otherwise, or where it is not applicable, like a problem involving only the picture of a graph or a table of numbers.
- Problem that ask you to "show that ... is true" or "justify your answer" or something like that, mean this: Give a clear and complete sequence of steps, each of which is obvious from an algebraic point of view, or which quotes one of our main results, like the Squeeze Theorem, or the Intermediate Value Theorem.

1. A calculator was used to produce the following table of numbers for $f(x) = \frac{1-\cos(x)}{x^2}$. Using the table, what is $\lim_{x\to 0} f(x)$?

x	$-.1$	$-.01$	0	.01	.1
$f(x)$	.49958	.499999999	ERROR	.4999999	.49958

2. Using the graph below of $f(x)$, find the following limits (or indicate that the limit does not exist):



(a) $\lim_{x\to 1} f(x)$

(b) $\lim_{x\to 3^+} f(x)$

(c) $\lim_{x\to 3^-} f(x)$

(d) $\lim_{x\to 3} f(x)$

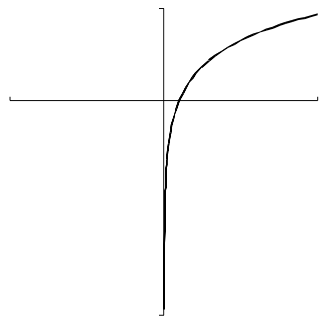
(e) $\lim_{x\to 5^+} f(x)$

(f) $\lim_{x\to 5^-} f(x)$

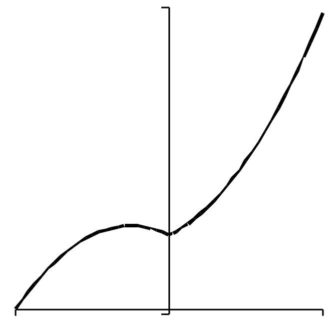
(g) $\lim_{x\to 7^-} f(x)$

3. Graphs (a)–(d) below are the graphs of four functions. Graphs I–IV are the graphs of their derivatives. For each function (a)–(d), indicate which of the graphs I-IV could be its derivative.

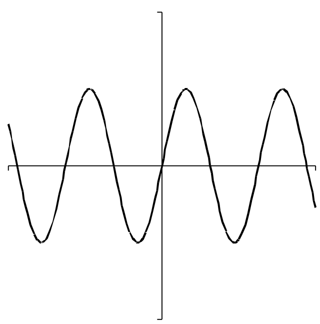
(a)



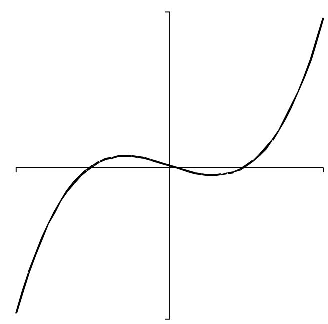
(b)



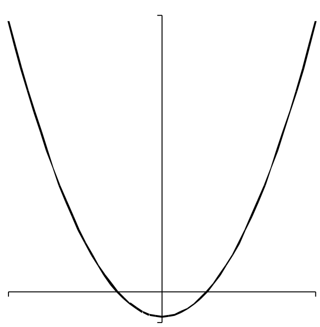
(c)



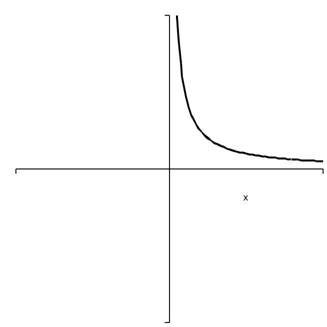
(d)



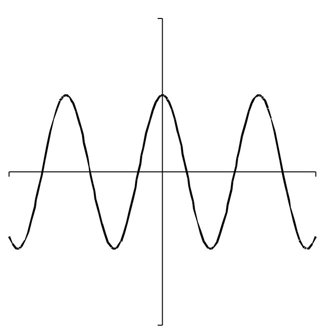
(I)



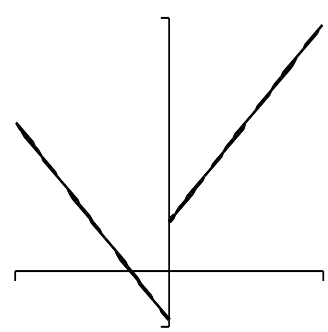
(II)



(III)



(IV)



4. Find $\lim_{x\to 0} x^4\cos(2/x)$. (Hint: use the squeeze theorem and the fact that $-1 \le \cos(\ ) \le 1$.)

5. Working only with integers, and without using your calculator, show that the equation $x^2 - 101 = 0$ has a solution, and give an interval $[a, b]$ which contains this solution.

   One way to think about this is that you are showing that $\sqrt{101}$ really exists, and you are giving an integer a which is too low, and another integer b which is too high. Here's what you should actually do.

   Show me an integer a which makes the function negative. Show me an integer b which makes the function positive. Quote the IMV (Intermediate Value Theorem) to give your conclusion.

6. Find the following (simple) limits

   (a) $\lim_{x\to 0} e^x\tan(x)$.

   (b) $\lim_{x\to 3} \dfrac{x^2 - x + 12}{x + 3}$.

7. Suppose a falling rock has position given by the following formula:

$$p(t) = -4.9t^2 + 13t + 10.$$

   Find a formula for the instantaneous velocity of the rock.

8. Find a such that the following function is continuous at $x = 0$ and justify your finding.

$$f(x) = \begin{cases} \sqrt{x} + a & \text{if } x \ge 0 \\ \sqrt{-x} + \cos(x) & \text{if } x < 0 \end{cases}$$

9. Solve the following equations for x. (This does not mean find a decimal solution. Instead, give an algebraic solution of x. For instance, you would write $x = \sqrt{2}$ not $x = 1.414$.)

   (a) $e^x = 7$

   (b) $\ln(x - 1) = 17$

   (c) $\pi x = 500/x^2$

   (d) $x^2 + 7x - 13 = 0$

   (e) $\sin(x) = -\sqrt{3}/2$ (find only one solution of x)

   (f) $\sin(x^2) = \frac{1}{2}$ (find only one solution of x)

10. Find the following derivatives:

    (a) $\dfrac{d}{dx} 10x^9$

    (b) $\dfrac{d}{dx}[-\sqrt{x} + 10x^9]$

    (c) $\dfrac{d}{dx}[\dfrac{1}{x} + \pi x^{3.14159}]$

    (d) $\dfrac{d}{dx}[x + x^2 + x^3 + x^4 + x^5 + e^x]$.

    (e) $\dfrac{d}{dx}[\sqrt{x}e^x]$

    (f) $\dfrac{d}{dx} \dfrac{10x^{7.3} - \sqrt{x}}{10e^x + x}$

11. Simplify $\dfrac{\frac{1}{x+h} - \frac{1}{x}}{h}$ as much as possible (note: I am *not* asking you to plug in $h = 0$ or to find the limit $\lim_{h\to 0}$ or to finish taking the derivative, etc. Just simplify.)

12. Using the definition, find the derivatives of the following functions, at the indicated point:

    (a) $f(x) = -5x^2 + 2x$, find $f'(3)$.

    (b) $f(x) = \frac{1}{x+1}$, find $f'(5)$.

    (c) $f(x) = \sqrt{2x + 1}$, find $f'(2)$.

13. Let $f(x) = x^3 - 2x^2 - 3x + 10$.

    (a) Find $f'(x)$.

    (b) Find the equation of the tangent line at $x = 1$.

14. Find the following limit, justify your answer by using a clear and complete sequnce of steps.

$$\lim_{x\to\infty} \frac{x + 2}{\sqrt{9x^2 + 1}}$$