Math 251, Spring 2005: Exam #2 Preview Problems

1. Using the definition of derivative find the derivative of the following functions:

   (a) $f(x) = e^x$. (Use the following $\lim_{h\to 0} \frac{e^h - 1}{h} = 1, \quad e^{x+h} = e^x e^h$.)

   (b) $f(x) = \frac{1}{x^2}$

2. Using implicit derivatives and the derivative of sin find/verify the shortcut derivative rule for $\sin^{-1}(x)$. (Hint: start with $y = \sin^{-1}(x)$, (a) solve this for x, (b) take the implicit derivative, solve for y', (c) to rewrite the formula so that you don't have y in the formula any more use a right triangle, make one of the angles y, and label the other two sides using the equation that you got as an answer to part (a).)

3. Find the derivative of the following functions:

   (a) $y = (x^4 - 3x^2 + 5)^3$

   (b) $y = \ln(\sec(2x+1)) - \frac{1}{2}\sin^2(x)$

   (c) $y = e^{-t}(t^2 - 2t + 2)$

   (d) $y = 2x\sqrt{x^2+1}$.

   (e) $y = (\cos(x))^x$

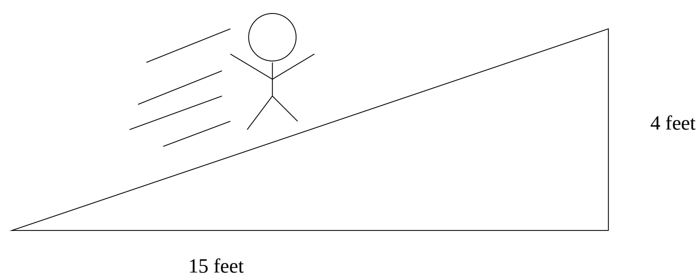
4. Suppose that $y = y(x)$ satisfies the equation $2xy - \ln y = 4$ and the condition.

   (a) Using implicit derivatives, calculate y' at the point $(2, 1)$.

   (b) Using your derivative from (a), find the equation of the tangent $L(x)$ at $(2, 1)$.

   (c) Using linear approximation, get an approximate value of $y(2.1)$.

5. A waterskier goes over a jump at a speed of 30 ft/s (this is her speed going along the diagonal). Find the speed at which she is rising (i.e. find the vertial speed) as she leaves the ramp.



6. If a tank holds 5000 gallons of water, which drains from the bottom of the tank in 40 minutes, then Torricelli's Law gives the volume V of water remaining in the tank after t minutes as

$$V = 5000\left(1 - \frac{t}{40}\right)^2 \qquad 0 \le t \le 40.$$

   Find the rat at which water is draining from the tank after (a) 5 min, (b) 10 min, (c) 20 min, and (d) 40 min. At which of these times is the water flowing the fastest? Which time is the water flowing the slowest?

7. Find the absolute minimum and maximum of the function $f(t) = t + \cot(t/2)$, on the interval $[\pi/4, 7\pi/4]$.

8. Let $f(x) = 1/x$ and let $g(x)$ be defined as

$$g(x) = \begin{cases} \dfrac{1}{x} & \text{if } x > 0 \\ \dfrac{1}{x} + 1 & \text{if } x < 0 \end{cases}$$

Show that $f'(x) = g'(x)$ for all x ("all" means all x where the functions are defined). Can we conclude from the corollary to the mean value theorem that $f(x) = g(x)$ for all x? Why or why not?

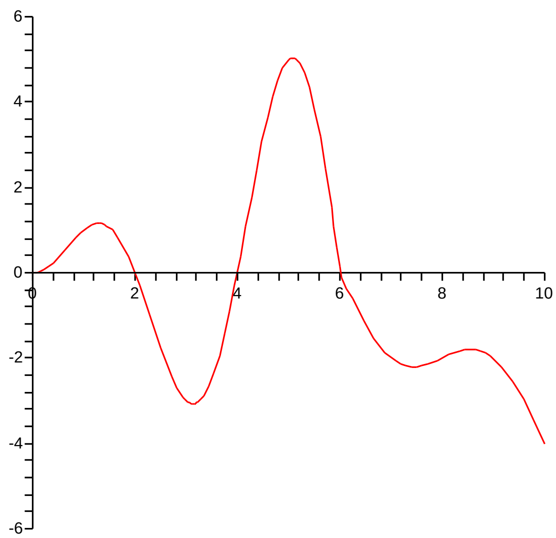
9. Let $f(x) = x^2 \ln(x)$.

(a) Find the critical points.

(b) Find the intervals of increase and decrease and identify each critical point as a local maximum, minimum or neither.

(c) Find the intervals of concavity.

10. The graph of the first derivative $f'(x)$ is shown below (the function doesn't exist to the left of zero, and to the right of 10 it just keeps going in the same direction that is shown).



(a) Find the critical points of $f(x)$.

(b) Find the intervals of increase and decrease of $f(x)$ and identify each critical point as a local maximum, minimum or neither.

(c) Find the intervals of concavity of $f(x)$.

11. Find the limit

$$\lim_{x \to \infty} \frac{\ln(\ln(x))}{x}$$