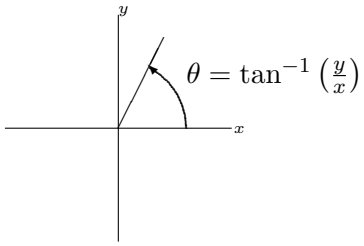
Issues with inverse tangent for Polar, Cylindrical and Spherical Coordinates
In Polar Coordinates (and thus in Cylindrical and Spherical Coordinates), we have the formula

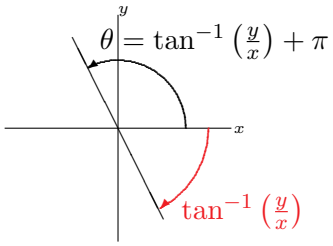
$$\tan\theta = \frac{y}{x}.$$

The angle θ needs to be between 0 and 2π radians, but $\tan^{-1}\frac{y}{x}$ gives us the angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. Thus just using $\tan^{-1}\frac{y}{x}$ only works if the angle is in QI of the xy-plane, i.e., if x and y are both positive. We need to adjust if our angle is in QII, QIII or QIV.

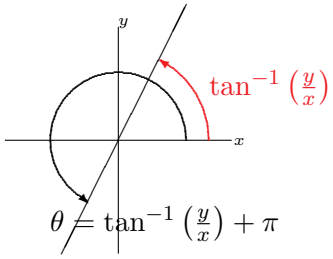
$\theta \in$ QI: $(x > 0,\, y > 0)$



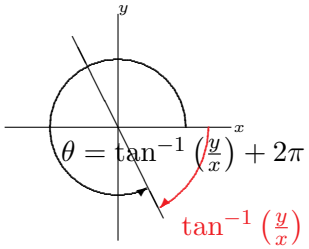
$\theta \in$ QII: $(x < 0,\, y > 0)$



$\theta \in$ QIII: $(x < 0,\, y < 0)$



$\theta \in$ QIV: $(x > 0,\, y < 0)$



Note: For spherical coordinates, $\phi \in [0, \pi]$ and if we use the formula $z = \rho\cos\phi$, $\cos^{-1}\left(\frac{z}{\rho}\right)$ will give us the correct angle. Thus there aren't any issues with ϕ as there are with θ.